Scaling Relations of Strike Slip Earthquakes With Different Slip Rate Dependent Properties at Depth

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Abstract

Empirical observations suggest that earthquake stress drop is generally constant. To investigate the effect of rupture width on earthquake scaling relations, we analyze synthetic seismicity produced by a 3D vertical strike slip fault model using two different profiles of frictional slip rate behavior below the seismogenic zone. Within the rate-and-state framework, a relatively abrupt transition of the $a - b$ profile from velocity weakening to strengthening at the base of the seismogenic crust produces increasing slip and stress drop with increasing event size. Choosing a smoother transition allows large earthquakes to propagate deeper, leading to similar slip-length scaling but constant stress drop scaling. Our numerical experiments support the idea that the maintenance of constant stress drop across the entire range of observed earthquake magnitudes may be achieved by allowing coseismic slip to rupture to depths below the seismogenic layer.

Introduction

Classically, earthquakes have been characterized as a dislocation with average slip $\bar{u}$ on a rectangular fault of width $W$ and length $L$ embedded in a homogeneous elastic half space with rigidity $\mu$, defining the seismic moment $M_o$ (Eq. 1). This dislocation results in an average stress drop $\Delta \tau$ on the fault plane [Kanamori and Anderson, 1975]

$$M_o = \mu LW \bar{u}$$
$$\Delta \tau = C\mu \frac{\bar{u}}{\mathcal{L}}$$

where $\mathcal{L}$ describes a characteristic length scale, and $C$ is a geometrical constant. It has been common practice since the work of Scholz [1982] to refer to the $L$ and $W$ models of earthquake behavior, whereby the length dimension $\mathcal{L}$ is assigned the value $L$ or $W$, respectively. Depending on which model is chosen, seismic moment is proportional to either $LW^2$ or $L^2W$

$$M_o = \alpha \mu LW^2 \quad W \text{ model}$$
$$M_o = \alpha \mu L^2W \quad L \text{ model}$$
with $\alpha = \Delta \tau / C$, and for small earthquakes, $L \approx W$, and hence $M_o \propto L^3$ [Scholz, 1997].

There is a long lasting debate to which is a more correct description of the earthquake process and thus the physical origin of the interrelationship between $M_o, L, W$ and $\alpha$ [Scholz, 1982; Romanowicz, 1992; Scholz, 1994; Bodin and Brune, 1996; Yin and Rogers, 1996; Matsuura and Sato, 1997; Mai and Beroza, 2000; Shaw and Scholz, 2001; Miller, 2002] with important implications to seismic hazard [Mai et al., 2007]. At the heart of the debate has been an aim to distill global mechanical dependencies beyond the complexity of individual earthquakes. Natural and synthetic area-moment scaling implies that the static stress drop of earthquakes of different magnitude is relatively constant [Kanamori and Anderson, 1975; Hanks, 1977; Wells and Coppersmith, 1994; Abercrombie, 1995; Rippeger et al., 2007], which has led to the interpretation that earthquakes are self-similar.

When the observation of constant stress drop is interpreted within the realm of elastic dislocation theory and the idea that coseismic slip is limited to the seismogenic layer, the observation of constant stress drop implies coseismic slip should scale with rupture width and thus ultimately saturate for ruptures of length dimension greater than the seismogenic thickness, $W_s$. However, observations relating slip (surface or on-fault) to rupture length $L$ generally show that coseismic displacement $\bar{u}$ continues to increase with $L$. The observations are thus in apparent conflict.

Discussing data by Scholz [1994], Shaw and Scholz [2001] posed a resolution to the apparent conflict through the numerical application of an elastodynamic model to show that the observed form of the coseismic slip versus length data could be explained with the application of scale invariant physics. Yet their results remained unsatisfactory when attempting to be explained in the context of a constant stress drop model for earthquakes limited to the seismogenic layer—the frictional velocity weakening part of the crust—of width $W_s$. Manighetti et al. [2007] have posed an alternate explanation that asserts that large earthquakes are the result of the rupture of multiple discrete segments, each characterized by constant stress drop. Interestingly, examining the latter analysis of the empirical data reveals a tendency for values of rupture width to increase with rupture length. Liu-Zeng et al. [2005] argue that constant stress drop is not necessarily required to explain slip-length scaling. Based on the observation that slip of individual earthquakes is complex [e.g. Mai and Beroza, 2002], Liu-Zeng et al. [2005] demonstrate that heterogeneous fault slip produced by pulse-like models of earthquake rupture can play a key role in slip-length scaling. Miller [2002] explains $\bar{u}$-$L$ data being controlled by the degree of overpressurization in a fault. In contrast to these models, empirical data do show that there is no apparent saturation of slip with $L$ [Wesnousky, 2007].

Thus, the observation of continued slip with rupture length is in conflict with independent observations indicating earthquakes’ stress drops are relatively constant continues to pose a paradoxical situation. Specifically, the assumption that $W \leq W_s$ together with the
increase in $\bar{u}$ with $L$ requires that large earthquakes have increasingly higher static stress drops than small events.

King and Wesnousky [2007] point out that because of depth dependent spatial resolution problems neither geodesy [Page et al., 2007] nor seismology [Beresnev, 2003] have yet ruled out the possibility that large earthquakes do produce coseismic slip beneath the seismogenic layer ($W > W_s \approx 15$ km). They in turn showed that the stress drop problem may be resolved by allowing coseismic slip to extend deeper into a zone with velocity strengthening properties [Tse and Rice, 1986]. While King and Wesnousky [2007] examined the problem in terms of a static dislocation model, here we use quasidynamic simulations of spatio-temporal evolution of slip to examine the effect of relaxing the condition that slip be constrained to zero at the base of the seismogenic layer. Specifically, we concern ourselves with the interrelations between $L, W, M_o, \bar{u}$ and $\Delta \tau$.

To do this synthetic seismicity is generated by a 3D quasidynamic rate-and-state controlled vertical strike slip fault model with imposed heterogeneity of a governing frictional parameter (Fig. 1). Two frictional slip rate dependent profiles are used: (1) A ‘box-like’ ($'b'$) profile similar to the one used in previous studies [Tse and Rice, 1986; Rice, 1993; Lapusta et al., 2000; Shibazaki, 2005; Hillers et al., 2006, 2007], which confines the depth extension of earthquakes to 15 km, and (2) a profile with a steeper ‘tapered’ ($'t'$) gradient below $z = -15$ km allowing accelerated slip associated with large events at greater depths while maintaining stable creep during interseismic periods (Fig. 2). Subsequent to briefly reviewing geological and seismological data bearing on the likelihood of a rate-dependent transition between stick-slip and stable sliding and the likelihood of coseismic slip beneath the seismogenic layer, we return to describe the models in more detail.

**Structure of Shear Zones and Likelihood of Deep Slip**

Mechanisms for deformation below continental strike slip fault zones revolve around the end member cases of (1) distributed deformation in the lower crust and upper mantle and (2) deformation concentrated on faults that cut across lower crust and mantle [e.g. Wilson et al., 2004]. Independent of structure, however, the ‘brittle-ductile’ transition, the mineral dependent peak in static crustal strength where Byerlee’s law and the viscous, temperature- and strain-rate-dependent strength of the lower crust intersect, is assumed to coincide with the lower bound of crustal seismicity. In a quartzofeldspathic crust, the transition is usually interpreted as the onset of quartz plasticity at 300°C ($\sim 11$ km), but feldspar shows plastic behavior at temperatures greater 450°C ($\sim 22$ km). Exhumed strike slip faults with large offsets indicate a narrow zone of shear localization in fault cores exhibiting cataclastic and pseudotachylitic material [Chester and Chester, 1998; Passchier and Trouw, 2005], whereas mylonites indicate narrow ductile shear zones at greater depths. Furthermore, the strength profile of mature strike slip faults may have a much smaller
strength gradient than the surrounding crust due to the presence of overpressurized fluids [Rice, 1992]. Dynamic models including thermo-mechanical localization feedbacks also lead to significantly altered strength profiles in the lower crust [Regenauer-Lieb et al., 2006]. Regenauer-Lieb and Yuen [2007] argue that brittle-ductile coupling, leading to a midcrustal core supporting most of the crust’s strength, offers new ways for understanding the root zone for large earthquakes, in contrast to purely frictional approaches. Discussing his now classic synoptic shear zone model, Scholz [1988] states that large earthquakes can propagate into the regime where quartz responses viscous, but he allows the maximum width to be limited to the 15 km depth range.

While distributed deformation is typical below strike slip environments at plate boundaries in oceanic regions [Molnar et al., 1999; Wilson et al., 2004], there is ample evidence from exhumed faults that deep portions of strike slip faults that undergo stable creep at low strain rates during interseismic periods show highly localized, rapid slip in response to high strain rates associated with downward propagating shear dislocations. By identifying brittlely deformed feldspars within a ductily deformed quartz matrix in an exhumed section of a strike slip fault corresponding to faulting depths of 15–20 km, Cole et al. [2007] modify Scholz’ model by assuming an even broader semi-brittle zone where coseismic localized slip and interseismic flow occur interchangeably [Sibson, 1980; Norris and Cooper, 2003]. Observations and models of instantaneous deepening and subsequent shallowing of seismicity after a mainshock [Schaff et al., 2002; Rolandone et al., 2004; Ben-Zion and Lyakhovsky, 2006] and variations in recurrence interval and moment of repeating aftershocks [Peng et al., 2005] are further evidence that the base of the seismogenic layer reflects a rate-dependent transition between stick-slip and stable sliding.

It is thus reasonable to suggest that large earthquakes are likely to penetrate significantly deeper into the crust on localized slip horizons than the hitherto assumed width of the seismogenic layer, \( W_s = 15 \) km, associated with the occurrence of background seismicity.

**Model Setup and Method**

We use a friction-based approach to simplify more complex, brittle-ductile feedback mechanisms in the semi-brittle transition zone at depth [Regenauer-Lieb et al., 2006] to test the implication of deep coseismic slip on earthquake scaling relations. We use the 3D strike slip fault model geometry discussed by Rice [1993] and Hillers et al. [2007], where a fault of \( X = [400, 300, 200] \) km by \( Z = -30 \) km is discretized into \( n_x = 1024 \) by \( n_z = [78, 102, 154] \) cells along strike and depth, respectively (Fig. 1), resulting in low, medium and high resolutions \( R^l, R^m, \) and \( R^h \). The fault is governed by the Dieterich-Ruina “slowness” rate-and-state formulation, and is loaded by a substrate below 30 km depth moving with \( v^\infty = 35 \) mm/year. This procedure greatly simplifies the numerical scheme. However, the implications of boundary conditions such as constant loading slip
rate or driving stress imposed at vertical faces of a 3D crustal block, or distributed drag at the base of the crustal layer, may be considered in an alternative modeling approach. Governing equations and the numerical procedure can be found in Hillers [2006] and Hillers et al. [2007]. We impose heterogeneous 2D distributions of the rate-and-state critical slip distance $D_c$ on the frictional interface to prevent the system from generating only system-wide events rupturing the entire fault [Ben-Zion, 1996; Heimpel, 2003; Shaw, 2004; Zöller et al., 2005].

To generate a database containing many large events, the degree of heterogeneity is chosen such that the system is allowed to produce big events more easily. Based on the results of Hillers et al. [2007], who investigated statistical properties of synthetic seismicity in response to $D_c$ maps parameterizing different degrees and types of heterogeneity, we use lognormal distributions of $D_c$ values across the fault with short correlation lengths along strike and depth (Fig. 1b, c). Using a default effective normal stress of $\sigma_e = 50 \text{ MPa}$ (Fig. 2a), minimum $D_c$ values are chosen appropriately to perform simulations in the continuum limit.

**Depth Dependent Properties**

Previous 2D antiplane [Tse and Rice, 1986; Lapusta et al., 2000] and 3D [Rice, 1993; Shibazaki, 2005; Hillers et al., 2006; Hillers and Miller, 2006, 2007] rate-and-state models of a strike slip fault adopted a temperature and hence depth dependent $a-b$ profile based on a geothermal gradient of the San Andreas fault [Stesky, 1975; Lachenbruch and Sass, 1980; Blanpied et al., 1991]. Tse and Rice [1986] demonstrated that this profile confines the occurrence of coseismic slip to about $z = -15 \text{ km}$, and Scholz [1988] related the $a-b$ rate dependence to the general structure of shear zones: Because of low normal stresses and the lack of hypocenters in this region, the topmost 3 km are usually interpreted to be velocity strengthening [Marone and Scholz, 1988; Marone et al., 1991], parameterized by $a > b$. The localized slip zone between $-3 < z < -15 \text{ km}$ has constant velocity weakening, $a < b$, allowing instabilities to nucleate. Slip rate dependence below this zone is again velocity strengthening, and generally assumed to correlate with the onset of quartz plasticity.

The approach taken by Tse and Rice [1986] focuses on the interpretation of frictional data, neglecting complex rheological properties not captured by this parameterization. As they note, “Depending on rheology, the depth estimated for transition from unstable to stable frictional slip on a deeply penetrating crustal fault may or may not be the same.” Various other approaches which further incorporated complex rheological properties have also been examined. Chester [1995] and Shibazaki et al. [2002] have shown that the transition from unstable to stable slip is not necessarily abrupt or time-independent.

To approximate the effect of different rheologies within the frictional framework, we vary
the rate of change of $a - b$ with depth, using two depth dependencies. The first shows
an abrupt transition at 15 km depth whereas in the second the value $a - b$ is taken
to smoothly increase with depth below 15 km (Figs. 2b & c). The latter profile allows
earthquakes to propagate significantly deeper, consistent with geological observations of
exhumed mature faults. Following the slip-depth distributions of King and Wesnousky
[2007] we compare scaling relations of events produced by these tapered distributions (‘t’
model) to properties of seismicity generated by the ‘standard’ box-like $a - b$ profile (‘b’
model) that confines coseismic slip to about 15 km depth. To highlight the difference be-
tween box and taper models we use a constant $a - b$ profile at shallow depths together
with a relatively abrupt velocity weakening to strengthening transition at 15 km depth for
the box models. We define the maximum depth extent of the velocity weakening region
where $a - b = -0.004 = \text{const.}$ to be the width of the seismogenic zone, $W_s$.

effective normal stress is chosen to be 50 MPa, $\sigma_e(z) = \sigma_n(z) - p(z)$, with $p(z) =
\max[p_{\text{hyd}}(z), \sigma_{\text{lith}}(z) - 50 \text{ MPa}]$ (Fig. 2a). Considering elevated pore pressures within the
fault zone alters the strength profile of crustal rocks associated with shear zone models
[Rice, 1992]. While the strength profile of intact rock and/or dry friction follows the Byer-
lee envelope, mature faults fail under much smaller shear stresses. To test the significance
of different degrees of overpressurization, we also conduct a set of simulations with an
increased effective normal stress using $\sigma_e = 75 \text{ MPa}$, $X = 300 \text{ km}$, $nx = 1024 \ (R^m)$, and
a properly adjusted $D_c$ value range.

Parameter Extraction

To extract slip events from the continuous stream of slip velocity, state, slip, and stress
data generated during the numerical experiments, we use a slightly modified procedure
discussed by Hillers et al. [2006] and Hillers et al. [2007]. We take the difference of slip
and stress of all cells between event initiation—the velocity of the hypocenter-cell is larger
than a trigger velocity $v_{\text{trs}}$, $v \geq v_{\text{trs}} = 10^4 \cdot v_{\infty}$—and event termination—all cells have
$v < v_{\text{trs}}$. We also monitor the maximum velocity of each cell during instabilities, $\hat{v}(x, z)$.
These original $u(x, z)$, $\Delta \tau(x, z)$, and $\bar{v}(x, z)$ distributions are interpolated on a $50 \times 50 \text{ m}^2$
grid, allowing the investigation of different choices of the definition of $I$, the subset of
cells that define the event. For example, we considered cells with $\hat{v}$ and $u$ being $[1, 10]\%$
of $v^*$ and $[2, 5, 10]\%$ of $u^*$ ($^*$ denotes maximum values of an array/population), respec-
tively, and find that these choices do not influence the overall conclusions presented here.
Results discussed in the present study were obtained considering cells with $u_i \geq 10\%$ of
$u^*$. Because of sometimes complex slip patterns consisting of multiple patches, we chose
only the largest contiguous patch from each event, to reduce the “chance in the game of
determining slip and length of an event” [Liu-Zeng et al., 2005]. The dimensions $L$ and
$W$ of those patches have been determined, as well as the mean slip $\bar{u}$ and mean stress
drop $\Delta \tau$. To estimate the event size in terms of seismological observables, we compute the
seismic potency \( P_i = a \sum_{i \in I} u_i \) or \( P = LW\bar{u} = A\bar{u} \) [Ben-Zion, 2003], with \( a \) being the area of a cell, and \( M_o = \mu P_i \), where \( \mu = 30 \) GPa. Figure 3 illustrates the relation between \( M_o \) and a local magnitude for southern California earthquakes \((1 < M_L < 7)\) [Ben-Zion and Zhu, 2002] and moment magnitude \( M_w \) [Kanamori, 1977].

While it is desirable to explore scaling relations up to very large earthquakes \((M_8)\), the largest magnitudes generated by our simulations are about \( M_7.5 \). A typical \( M_7.5 \) event from the box model has \( W = 15 \) km, \( L = 250 \) km, and \( \bar{u} = 220 \) cm, which leads to \( P = 8.3 \cdot 10^5 \) cm km\(^2\). The corresponding potency of an \( M_8 \) event is \( P = 10^7 \) cm km\(^2\), which requires \( W = 15 \) km, \( L = 670 \) km, and \( \bar{u} = 10 \) m. The dimensions and resolutions necessary to produce (several) model quakes of this size are at present computationally too cumbersome.

**Analysis**

**Slip & Stress Drop Distributions of Typical Events**

Figure 4a-c shows some typical slip and corresponding stress drop distributions for events with \( M_o \approx [0.2, 1, 5] \cdot 10^{18} \) Nm generated by a high resolution box model. Figures 4a, b, e demonstrate the effect of differing the values of defining limits \( I \), using thresholds of \([2, 5, 10]\)% of \( u^* \). While the effect is largest for small events because \( u^* \) is relatively small here, the choice of the threshold is less significant for events with \( M_o > 5 \cdot 10^{18} \) Nm. This is illustrated by the close proximity of the contours for the larger event (Fig. 4c) compared to the significantly different contour pattern shown in 4a. Figure 4c show an event where \( W_b \) reaches the width of the seismogenic zone, \( W_s \). Hence, in our analysis, we will focus on large events with moments above that threshold, corresponding to \( M_L 6.5 \) and \( M_w 6.4 \), respectively (Fig. 3).

Figures 4d & e shows two example stress change distributions of small events to illustrate the origin of scatter for small moments in subsequently discussed scaling relations. Small compact shapes (4d) result in stress drops that are in the range of the constant \( \Delta\tau \) level for larger events. Irregular shapes (4e) obtained by the same event extraction algorithm result in lower average stress drop. Small \( \Delta\tau \) values for small earthquakes are a result of the heterogeneous boundary conditions and of the quasidynamic approximation, because maximum slip rates are considerably smaller \((v^* \approx v^{trs})\) than for large events \((v^* \gg v^{trs})\). In tandem with the relatively coarse resolution and the event-extraction method, this leads to stress drops that underestimate observed values. Strictly speaking, these events are not mature slip events, but instabilities that are frustrated at early stages when slip rates have not become ‘really dynamic’ yet.

**Main Result**
Figure 5 shows a compilation of scaling relations between $L, W, M_o, \bar{u}$ and $\Delta \tau$ obtained from a typical box and taper model with medium resolution ($R^m$). As explained, data for small events ($< M_w 6.4$) show considerable scatter, whereas data for event sizes with $W \geq W_s$ do not.

Spatial Dimension, $L$ and $W$ ($5a \& d$): Both box and taper models show identical $L - M_o$ scaling, i.e., similar rupture lengths (which are most easy to determine for real ruptures) result in seismic moments of comparable size. For this reason, it is unlikely that empirical observations of moment versus length, either rupture length [Wells and Coppersmith, 1994] or the length of aftershock zones [e.g. Kagan, 2002], are sufficient to lend any insight to the depth extent of earthquakes. Because taper $a - b$ profiles allow ruptures to release strain energy by propagating deeper, rupture length as a function of moment tends to be larger for box events than taper events, hence $L^*_{b} > L^*_{t}$. We observe a break in $L_b - M_{ob}$ scaling at $M_o = 2 \cdot 10^{19}$ Nm ($M_w 6.6$), suggesting a breakdown of self-similarity associated with the saturation of $W_s$, i.e., box ruptures tend to grow exclusively in along-strike direction. The few datapoints for the taper model make it difficult to infer the same dependence. Rupture width $W_t$ reaches a maximum at about $M_o \approx 2 \cdot 10^{19}$ Nm, due to the imposed $a - b$ profile prohibiting unstable slip below $z = -15$ km. The tapered $a - b$ profiles allow $W_t$ to grow with increasing $L_t$, and to level off at greater depths, depending on the gradient of the profile (see below).

Slip ($5b, c \& e$): Slip-length scaling is at the heart of the problem whether $\Delta \tau$ is expected to be constant or not. Our synthetic data up to $L_b = 250$ km suggests a continuous increase with slip, but decreasing growth rate, consistent with King and Wesnousky [2007]. Mean slip of large taper events, $\bar{u}_t$, tends to be less than corresponding $\bar{u}_b$, because the area of taper events is systematically larger, maintaining the observed constant $L - M_o$ scaling.

Stress Drop ($5c \& f$): The most significant result is the increase of $\Delta \tau$ with $\bar{u}$ and $M_o$ for box events, whereas large earthquakes from taper models do not show an increase of $\Delta \tau$ with $\bar{u}$ or $M_o$. Only at the largest moments where $W_t$ begins to saturate, datapoints allow an orientation towards larger $\Delta \tau$ values. Hence, the continued downward propagation of taper events leads to constant stress drops for large events, compatible with seismic observations. Furthermore, identical $L_t -$ and $L_b - M_o$ scaling suggests that differences in scaling behavior can not be detected by length measurements, but depend on an accurate estimate of the rupture’s depth extension.

Area-moment: Figure 6 shows the corresponding area-moment scaling of the box and taper model discussed in Figure 5. At first order, the bulk of data follows the expected linear relation on a double logarithmic scale associated with a classic constant stress drop.
model. However, interesting deviations from this global trend are as follows:

1. In both cases, $\Delta \tau$ of small earthquakes (6b, e) increase with smaller $A$ (here: $A = \sum_{i \in I} a_i$) for constant $M_o$ following the theoretical lines of constant stress drop (6b & e), consistent with dislocation theory, since for $\bar{u}_1 = \bar{u}_2$ and $A_1 < A_2$ follows $\Delta \tau_1 > \Delta \tau_2$.

2. The increase in circle-size illustrates the increase in $\Delta \tau$ for box model events with increasing moment which is not observed in the taper model (6c & f, as in 5f). That is, earthquakes with $W > W_s$ produce scaling relations that are fully compatible with observed $A - M_o$ relations and theoretical constant $\Delta \tau$ considerations, in contrast to slip events with $W = W_s$.

Comparison of Two Large Box and Taper Events With Similar Magnitude

Figures 7a & d show final slip distributions of two events with comparable moments from the previously discussed box and taper event populations ($R^m$), respectively, illustrating the origin of the discussed differences in scaling relations for large earthquakes. Data for these particular earthquakes are marked by the enlarged plus sign and circle, respectively, throughout Figure 5. Table 1 gives a complete overview of the inferred parameters, showing that both lengths and moments are approximately the same. Due to different $a - b$ dependencies at depth, $W_t$ is 20% larger than $W_b$, but the mean slip for this pair is of comparable size (8% difference). Consequently, the box event has more slip on a smaller area and thus an about 20% larger average stress drop than the taper event. Slip below $W_s$ of the box event contributes only 3% to the total moment, in contrast to the 13% seismic moment release by the taper event.

The corresponding figures showing potency release contours (7b & e) reveal the effect of the abrupt $a - b$ transition of the box model: Slip associated with the instability does not propagate below $z \approx -16$ km, and continues to grow only laterally after the event has reached about one third its final length. The taper event shows a different behavior: Slip accumulates continuously along strike and depth, but with significantly reduced growth rates and slip velocities at depth. The relative decrease in rupture speed and slip rate below the seismogenic layer for the taper model suggests that combined inversions of high- and low-frequency bands are necessary to resolve the actual amount of moment release.

The systematic increase in depth and slip with $L$ cannot be explained solely by a pulse ($W$-) model, but requires there be some continued slip behind the propagating rupture front. Similarly, the relaxation of a maximum rupture width $W^* \geq W_s$ violates the fundamental $L$-model assumption. Hence, the behavior exhibited by taper model earthquakes can at best be described by a ‘hybrid $L - W$’ scaling behavior, demonstrating that neither $L$ or $W$ exclusively control the final size of an instability. Maximum slip velocities during the box event show a sharp transition around $W_s$ (7c), whereas increased velocities of the
taper event are tapered below $z = -14 \text{ km}$ ($7\ell$).

### Effect of Steeper $a-b$ Gradient $t2$

A steeper gradient $t2$ in Figure 2c affects $W_t - M_o$ scaling most significantly (Fig. 8a), allowing ruptures to propagate deeper compared to the $t1$ gradient. Note that the maximum width using $t2$ is about $W_{t2}^* = 22 \text{ km}$, whereas $W_{t1}^* = 19 \text{ km}$ (Fig. 5d). This increase in rupture area confirms that average slip generated by taper models is less than slip of box models (8b). Furthermore, gradient $t2$ leads to $M_{st}^* \approx M_{ob}^*$, showing no increase of $\Delta\tau_t$ at largest event sizes (8c), verifying the separation in stress drop scaling between taper and box event populations. Or, more specifically, the lack of dependence of stress drop on rupture length for large earthquakes.

### Effect of Event Size Threshold

Using a smaller relative slip threshold ($\%$-of-$u^*$) leads to a larger subset $I$, increasing the number of cells with small slip values (Fig. 4a). Hence, the scatter in $W$ and $L$ increases for small events but the corresponding estimates for large events do not change (Fig. 7), $\bar{u}$ scaling remains unaffected, and the $\Delta\tau$ scatter at small events is reduced together with a slight decrease of the overall $\Delta\tau$ level.

### Effect of Trigger Threshold

A larger trigger threshold $v_{trs} = 10^6 \cdot v^\infty$ instead of $v_{trs} = 10^4 \cdot v^\infty$ decreases the number of cells belonging to a given event. Due to the localized acceleration at event nucleation, the consideration of slip accumulated with lower slip rates corresponds exclusively to the inclusion of ‘afterslip’ like displacement, which takes place at the edges of the fault (Figs. 7c & f). Hence, taper events do not reach as deep as using the default $v_{trs}$ value (Fig. 9a), which results in a decrease in rupture area. Similar displacements (9b) on a reduced area lead to a resulting higher but still constant $\Delta\tau$ level (9c). With a larger $v_{trs}$ value the differences between taper and box scaling (Figs. 9d-f) are less pronounced. While the widths for large events still belong to different populations, the $\bar{u} - L$ scaling suggests only a weak trend of separation. The $\Delta\tau - M_o$ data are less significantly separated, although the increasing and constant trend in the box and taper models, respectively, is observable. Differences in stress drop behavior between the two datasets can be detected only for the largest events.

### Effect of Normal Stress

To investigate the impact of different effective normal stress conditions, we compare a box and taper model set with $\sigma_e = 75 \text{ MPa}$ to the set using the default value of $\sigma_e = 50 \text{ MPa}$ ($R_m$). Figures 10a-c compare the results of two taper models with different pore pressure profiles. While the $W - M_o$ scaling with $\sigma_e = 75 \text{ MPa}$ shows slightly smaller widths for the same moment (10a), coseismic displacements tend to be significantly larger than for
\[ \sigma_e = 50 \text{ MPa} \text{ (10b). Consequently, } \Delta \tau \text{ values are at a higher level, but exhibit the same constant trend (10c). Increased fault strength leads to larger slip deficits and in turn to increasing coseismic slip values and a reduced seismicity rate.} \]

Scaling relations of box and taper models with \( \sigma_e = 75 \text{ MPa} \text{ (10d–f) show the previously established features, where a separation of } \bar{u} - L \text{ data from the two models occurs at larger } L \text{ values (100 km) compared to the } \sigma_e = 50 \text{ MPa case (Fig. 5b, 75 km). Stress drops } \Delta \tau_t \text{ for large } M_{ot} \text{ tends to increase because of larger displacements at associated widths compared to } \sigma_e = 50 \text{ MPa (10f).} \]

**Effect of Resolution—Compliance**

It is expected that a higher spatio-temporal resolution (allowing to use smaller \( D_c \) values) leads to a more pronounced difference between the box and taper case, similar to the use of a steeper taper gradient discussed in Figure 8. However, the limited spatial dimension \( X = 200 \text{ km for } R^b \text{ (Fig. 11c) leads to rupture lengths and hence maximum event sizes that are about half a magnitude smaller than for the } R^t \text{ case (11a). Nevertheless, the following trends can be extracted: (1) The resolution does not affect the maximum depth extension, } \tilde{W}^* \text{ (11 left). (2) The separation of } \bar{u} - L \text{ populations becomes effective at smaller lengths for more compliant grids (11 center). (3) Consequently, differences of the stress drop levels at the transition between small and large events become more pronounced the higher the resolution (11 right).} \]

These trends can be attributed to the compliance of the grid, the associated choice of minimum \( D_c \), and the resulting slip rate dependence with size (Fig. 12). Small events produced by low resolution models have larger velocities because larger regions have to become unstable to reach the trigger threshold. Stiffer numerical grids lead to consistently smaller slip velocities \( v^* \) during medium sized slip events. These differences do not persist at large, fully developed instabilities.

The data for large \( L \) in the \( \bar{u} - L \) scaling figure 11a \( (L_b \approx 300 \text{ km, } L_t \approx 260 \text{ km}) \) suggest a final saturation of \( \bar{u} \) for large events, most visible for the box case. This can readily be explained by the saturation of \( W \) at about \( W_s \). But even in the taper case, the largest event saturates at the maximum depth allowed by the applied \( a - b \) profile \( (11a, W - M_o \text{ scaling}). \) Hence, saturation of \( \bar{u} \) for large \( L \) is demonstrated to be associated with the saturation of \( W \). The lack of available data suggesting this behavior for natural seismicity thus supports the hypothesis made in this study.

**Slip-Depth and Hypocenter-Depth Distribution**

Slip-depth distributions further illustrate the effect of spatio-temporal resolution (Fig. 13). On average, large \( (> M7) \) earthquakes accumulate more slip the better the resolution, but
cease at identical depths (13a & c). Slip in box events is consistently larger than in taper events (5b), while for the latter slip horizons propagate significantly deeper (5d, 8a, 9d, 10d, 11 left), leading to a significant difference in moment partition between above and below $W_s$, respectively (see figure caption for details).

The inferred hypocenter-depth distributions support usage of the term ‘seismogenic zone’ in the present context (13b & d). Events nucleate exclusively in regions of negative $a - b$ above 15 km depth. However, the generated distribution, with the peak of large event initiation around $z = -5$ km is at odds with the observations that large events nucleate lower and near the brittle-ductile transition [Scholz, 1988]. This inconsistency can be explained with characteristics of the model, related to the manner in which the fault model is loaded [Tse and Rice, 1986]. The similar shape of the corresponding slip- and hypocenter-depth distributions shows the negligible effect of the topmost velocity-strengthening part in the tapered $a - b$ profile, however, the peak in the taper case is about 2 kms deeper.

**Conclusion**

Heterogeneous fault properties along strike and depth, and the resulting complex rupture propagation, heterogeneous slip distributions, and spatially variable stress drops of large events remind us that the results presented in this study are a first order approximation to properties of real seismicity. Nevertheless, based on an increasing body of evidence suggesting the occurrence of coseismic slip on localized structure below the seismogenic zone defined by background seismicity, we used a rate-and-state 3D strike slip fault framework with a modified slip rate dependent profile at depth to explore the effect of rupture width on earthquake scaling relations. This procedure captures complex feedback mechanisms within the broad semi-brittle mid-crust region and produces synthetic data that are compatible with observations, including spatially complex slip and stress drop distributions.

We performed systematic numerical experiments to investigate the effect of the $a - b$ gradient, trigger threshold, effective normal stress, and spatio-temporal resolution. The results show that earthquake populations confined to $W_s = 15$ km produce slip-length relations compatible with observations, but show a linear dependence of stress drop on earthquake size, which is not supported by observation. In contrast, synthetic seismicity generated by a fault model with a less restrictive slip rate dependence below $W_s$ has larger rupture widths, consistently smaller average slip values and consequently constant stress drop scaling.

Event sizes and thus scaling relations have been demonstrated to be sensitive to the particular definition of coseismic slip rates, as a result of the quasidynamic character of the numerical experiments. The numerical experiments presented here, by Das [1982], and more recently by Shaw and Wesnousky [2007] demonstrate that seismic slip rates and scal-
ing laws are sensitive to the boundary conditions assumed at the base of the seismogenic layer. The buried fault model of Das [1982] assumed deformation above and below the seismogenic layer to be ductile and slip to occur in the absence of stress-drop or seismic radiation. The results presented here and by Shaw and Wesnousky [2007] are effectively governed by friction laws for which coseismic slip below the seismogenic layer does produce stress drop and seismic radiation. Regardless of the variations in the approaches to modeling the problem, the results show that the occurrence of coseismic slip below the seismogenic layer is a physically plausible explanation of slip versus length scaling laws.

King and Wesnousky [2007] discussed the ability of coseismic slip to propagate below \( W_s \) in the context of fault zone (im-) maturity and geometric heterogeneity. Hillers et al. [2007] parameterized faults at different evolutionary stages using 2D \( D_l \) distributions with different degrees and types of heterogeneity. It is thus reasonable to investigate the co-evolution of fault zone complexity [Wesnousky, 1988] and properties at depth controlling deep slip simultaneously.

The decreased growth rates and slip velocities below \( W_s \) of synthetic earthquakes with deep slip are compatible with observations that substantial portions of slow slip might not be detected in the frequency band which is sensitive to rapid slip in the seismogenic part, but at significantly longer periods [e.g. Stein and Okal, 2005]. Together with the current depth resolution of geodetic and seismological observations, the results presented here might encourage more rigorous search for deep coseismic slip in nature. In the event that the model results are shown to be a good approximation of fault behavior, the characterization of earthquake dynamics and associated scaling properties should leave behind the discussion of fault rupture processes in terms of \( L \)-or-\( W \) scaling behavior.
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References


Table 1: Source parameters for events discussed in Figures 4 and 7. The detailed area $\sum_{i \in I} a_i$ is obtained considering individual cells in the subset $I$, with $a$ being the cell size. $P_i$ is the detailed potency, while $P$ is determined using averaging $L$ and $W$ values. Differences between $P_i$ and $P$ are due to complex shapes—most abundant for small events, but less pronounced for large events. Read 1.93e+17 as $1.93 \cdot 10^{17}$.
Figure 1: (a) Model geometry. (b) Example 2D $D_c$ distribution with short (2 km) correlation lengths along strike and depth from a case with lowest resolution, $R_l$. $X = 400$ km, $Z = -30$ km, $nx = 1024$, $nz = 78$. (c) Corresponding lognormal distribution of $D_c$ values across the fault plane. This parameterization has been chosen to favor the occurrence of large events. See Hillers et al. [2007] for the role of $D_c$ distributions as a tuning parameter.
Figure 2: (a) Lithostatic normal stress ($\sigma_n$), pore pressure ($p$) and effective normal stress ($\sigma_e$). (b) Box (‘b’) and (c) two tapered (‘t’) $a-b$ profiles used throughout the study. Each $a-b$ profile (b, t1) is used respectively with a $D_i$ map (e.g. Fig 1b). The seismogenic width $W_s$ is defined to be the deepest point where $a-b = -0.004 = \text{const.}$, i.e., $z = -14 \text{ km}$. 
Figure 3: Local magnitude for Southern California $M_L$ after Ben-Zion and Zhu [2002], and moment magnitude $M_w$ after Kanamori [1977] versus seismic moment. The dotted segment of $M_L$ indicates that $M_L$ has been determined for $1 < M_L < 7$. 
Figure 4: Example slip maps and corresponding stress change patterns produced by a box model (resolution $R^b$) to illustrate the saturation of $W_s$ (a-e) and the origin of scatter in stress drop values for small events (d-e), respectively. Contours of increasing thickness enclose areas of slip with [2, 5, 10]% of the maximum slip value, $u^*$. Thickest line indicates the governing 10%-of-$u^*$ contour. White circles denote hypocenters determined at the time of triggering within a circular region centered at the fastest slipping cell [Hillers et al., 2007]. (a)–(c): Due to the applied $a-b$ profile, events with $M_o > 5 \cdot 10^{18}$ Nm ($M_L 6.5$, Fig. 3) propagate only along strike and mark the approximate transition between small and large events. (d)–(e): Slip and stress drop distributions for two $M_L 5.5$ earthquakes, where $\Delta \tau$ differs about 300%. See Table 1 for detailed summary of event properties.
Figure 5: Scaling relations of a box and taper model with $\sigma_0 = 50$ MPa, $v^{\text{trs}} = 10^4 \cdot v^\infty$, $R^m$. (a) Length-moment, (b) Slip-length, (c) Stress drop-slip, (d) Width-moment, (e) Slip-moment, (f) Stress drop-moment scaling. Here and in all subsequent scaling figures, if not specified otherwise: Black plus signs: Taper model. Grey circles: Box model. Dotted horizontal and vertical lines separate small from large events (Fig. 4). Highlighted symbols denote events discussed in Figure 7. Key results: (a) Events from the box and taper populations with similar moments cannot be distinguished by their lengths; (b) Nonlinear $\bar{u} - L$ scaling as discussed in King and Wesnousky [2007]; (d) Relaxed $a-b$ profiles below $W_s$ result in increased widths of taper events; (f) For large events, $\Delta \tau$ of box events scales with $M_o$, in contrast to observations, whereas the stress drop of taper events does not scale with event size.
Figure 6: Area-moment scaling with $A = \sum_{i \in I} a_i$. Top: Data from box model (Fig. 5, grey symbols). Bottom: Data from taper model (Fig. 5, black symbols). (a) & (d) Entire magnitude range. (b) & (e) Subset of a\&d, $A - M_o$ scaling for small events. (c) & (f) Subset of a\&d, $A - M_o$ scaling for large events. Broken lines denote theoretical estimates of constant stress drop. Note that for profile $t1$, $\Delta \tau$ of individual events, symbolized by the circle size, follows the constant theoretical estimates for large events, whereas for profile $b$ the measured stress drops increase with increasing moment.
Figure 7: Comparison of two respective $M_L7.3$ events (highlighted in Fig. 5) from the (a)–(c) box and (d)–(f) taper model, $R^m$, with $L_b \approx L_t$ and $M_{ob} \approx M_{ot}$. (a) & (d) Final slip distributions. (b) & (e) Contours of constant potency release. (First ten contours are plotted at a five times higher rate than subsequent contours.) Note the difference in upward propagation at early stages due to $a > b$ vs. $a < b$ in the topmost part of the box and taper model, respectively (Fig. 2). (c) & (f) Distributions of maximum slip rate values during the events, scaled to the load velocity $v^\infty$. The growth and velocity images of the taper event (e & f) suggest that growth rates and to a lesser extent slip rates below $W_s$ are smaller compared to values in the seismogenic zone. However, moment release above/below $W_s$ is 87/13% in the taper model, compared to 97/3% in the box model. See Table 1 for detailed summary of event properties.
Figure 8: Effect of steeper $a-b$ taper-gradient (profile $t2$, Fig. 2c), $R_m$. (a) Width-moment, (b) Slip-length, (c) Stress drop-moment scaling. Box data (grey) as in Fig. 5. Note the increased widths of large events compared to the data produced with the $t1$ profile (5, black data), and the resulting clearer separation of $\Delta \tau - M_o$ data at large magnitudes.
Figure 9: Effect of different trigger threshold, \( v^{\text{trs}} \). (a)–(c) Scaling relations of two taper models with \( v^{\text{trs}} = 10^4 \cdot v^\infty \) (‘+’ as in Fig. 5) and \( v^{\text{trs}} = 10^6 \cdot v^\infty \). Most significant is the constant shift in \( \Delta \tau-M_o \) scaling. (d)–(f) Comparison of box and taper event population, both with \( v^{\text{trs}} = 10^6 \cdot v^\infty \), \( R^m \). Triangles as in a–c.
Figure 10: Effect of different normal stress $\sigma_e$ (Fig. 2a). (a)–(c) Scaling relations of two taper models with $\sigma_e = 50$ MPa (‘+’ as in Figs. 5, 9a–c) and $\sigma_e = 75$ MPa. Most significant is the large increase in coseismic slip occurring on the stronger fault, and the associated constant shift in $\Delta \tau$-$M_o$ scaling. (d)–(f) Comparison of box and taper event population, both with $\sigma_e = 75$ MPa, $R''$. Triangles as in a–c.
Figure 11: Effect of compliance ($X/nx \rightarrow \min[D_i]$). (a) Low resolution $R^l$. Event with largest moment in $W_t - M_{ot}$ scaling corresponds to the $L_t = 263$ km event in $\bar{u}_t - L_t$ scaling. Note the saturation of its width. (b) Medium resolution $R^m$. (c) High resolution $R^h$. The longest $R^l$ models produce the largest synthetic earthquakes. The overall similarity of the scaling relations among the three cases suggests the first order results are insensitive to resolution and hence robust features of the physical parameterization.
Figure 12: Effect of compliance on maximum velocity $v^*$. Small instabilities produced by less compliant models have larger velocities because larger regions have to accelerate, as a consequence of the $\min[D_c]$-dependent nucleation size. Medium sized events from high resolution simulations show larger velocities, because smaller regions can become more unstable. Data from all resolutions merge at large magnitudes. $v^{\text{trs}} = 10^4 \cdot v^{\infty}$, $\sigma_e = 50$ MPa.
Figure 13: Average slip-depth profiles along strike for $> M_L$7 earthquakes from (a) box and (c) taper models. Solid lines: $\sigma_e = 50$ MPa, $v^m = 10^4 \cdot v^\infty$, taper models used $a-b$ profile $t1$. Dash-dotted lines: $\sigma_e = 75$ MPa, $R^m$. Dashed line: $a-b$ profile $t2$, $R^m$. Relative moment release above/below $W_s$ for the box models: $R^l 96/4\%$, $R^m 97/3\%$, $R^h 96/4\%$, $\sigma_e = 75$ MPa 97/3\%. Taper models: $R^l 86/14\%$, $R^m 88/12\%$, $R^h 87/13\%$, $\sigma_e = 75$ MPa 88/12\%, $t2$ 80/20\%. (b) & (d) Hypocenter (seismicity) distribution with depth (all event sizes). White squares show number of corresponding hypocenters for $> M_L$7 events, multiplied by five for clarity. Note different scales on the respective horizontal axes. The peak in the number of hypocenters for the taper models is lowered because of the strengthening effect of the tapered $a-b$ profile at shallow depth.