Fault Scaling Relationships Depend on the Average Geological Slip Rate

John G. Anderson, Glenn P. Biasi, Steven G. Wesnousky

Abstract

This study addresses whether knowing the geological slip rates on a fault improves estimates of magnitude ($M_W$) of shallow, continental surface-rupturing earthquakes. Based on 43 earthquakes from the database of Wells and Coppersmith (1994), Anderson et al. (1996) previously suggested that estimates of $M_W$ from rupture length ($L_E$) are improved by incorporating the slip rate of the fault ($S_F$). We re-evaluate this relationship with an expanded database of 80 events, that includes 57 strike-slip, 12 reverse, and 11 normal faulting events. When the data are subdivided by fault mechanism, magnitude predictions from rupture length are improved for strike-slip faults when slip rate is included, but not for reverse or normal faults. Whether or not the slip rate term is present, a linear model with $M_W \sim \log L_E$ over all rupture lengths implies that the stress drop depends on rupture length - an observation that is not supported by teleseismic observations. We consider two other models, including one we prefer because it has constant stress drop over the entire range of $L_E$ for any constant value of $S_F$ and fits the data as well as the linear model. The dependence on slip rate for strike-slip faults is a persistent feature of all considered models. The observed dependence on $S_F$ supports the conclusion that for strike-slip faults of a given length, the static stress drop, on average, tends to decrease as the fault slip rate increases.
Introduction

Models for estimating the possible magnitude of an earthquake from geological observations of the fault length are an essential component of any state-of-the-art seismic hazard analysis. The input to either a probabilistic or deterministic seismic hazard analysis requires geological constraints because the duration of instrumental observations of seismicity are too short to observe the size and estimate the occurrence rates of the largest earthquakes (e.g. Allen, 1975; Wesnousky et al., 1983 ). Thus, wherever evidence in the geological record suggests earthquake activity, it is essential for the seismic hazard analysis to consider the hazard from that fault, and an estimate of the magnitude of the earthquake that might occur on the fault is an essential part of the process. The primary goal of this study is to determine if magnitude estimates that are commonly estimated from fault length can be improved by incorporating the geological slip rate of the fault.

Numerous models for estimating magnitude from rupture length have been published. Early studies were by Tocher (1958) and Iida (1959). Wells and Coppersmith (1994) published an extensive scaling study based on 244 earthquakes. Some of the more recent studies include Anderson et al., (1996), Hanks and Bakun (2002, 2008), Shaw and Wesnousky (2008), Blaser et al. (2010), Strasser et al. (2010), and Leonard (2010, 2012, 2014). For probabilistic studies, and for earthquake source physics, it is valuable to try to reduce the uncertainty in these relations. Anderson et al. (1996) (hereafter abbreviated as AWS96) investigated whether including the slip rate on a fault improves magnitude estimates given rupture length. They found that it does, and proposed the relationship $M_W = 5.12 + 1.16 \log L_E - 0.20 \log S_F$, thus indicating that slip rate is a factor. A physical interpretation of a significant dependence on slip rate is that, for a common rupture length, faults with higher slip rates tend to have smaller static
stress drop. Since the publication of AWS96, the number of earthquakes with available magnitude, rupture length, and slip rate estimates has approximately doubled. This paper considers whether these new data improve or modify the conclusions from the earlier study.

One consideration in developing a scaling model is that seismological observations have found stress drop in earthquakes to be practically independent of magnitude. Kanamori and Anderson (1975) is one of the early papers to make this observation. Recent studies that have supported this result include Allman and Shearer (2009) and Baltay et al. (2011). Apparent exceptions have been reported based on Fourier spectra of smaller earthquakes, but as magnitude decreases, attenuation can cause spectral shapes to behave the same as they would for decreasing stress drop (e.g. Anderson, 1986). Studies that have taken considerable care to separate these effects have generally concluded that the average stress drop remains independent of magnitude down to extremely small magnitudes (e.g. Abercrombie, 1995; Ide et al., 2003; Baltay et al., 2010, 2011). However all of these studies find that for any given fault dimension the range of magnitudes can vary considerably (e.g. Kanamori and Allen, 1986). In spite of this variability, it seems reasonable to evaluate a scaling relationship that is based on a constant stress drop before considering the additional effect of the fault slip rate. This vision guides the development of the considered scaling relationships. Details of these models for the relationship of stress drop and the fault dimensions are deferred to the Appendix. The following sections describe the data, present the summary equations for three alternative models, fit the alternative models to the data, and discuss the results.
Data

Tables 1 and 2 list the earthquakes used in this analysis. A complete list of considered events is given in the spreadsheet *aMasterEventTable_2015-10-6b.xlsx*. This spreadsheet, together with citations for all values, is given in an Online Supplement. Tables 1 and 2 give only the preferred estimates of $M_W$, $L_E$, and $S_F$. The corresponding uncertainty ranges used in the analysis are given in the online supplement. Events considered for analysis come from Biasi and Wesnousky (2016) and Anderson et al. (1996). Some events they considered were not included because they did not have surface rupture or because fault slip rate was not sufficiently well known.

Figure 1 shows the cumulative number of earthquakes used as a function of time. From 1944-2013, the rate of usable events is relatively steady, about 0.9 events per year. The rate is lower prior to ~1930 suggesting that the earlier historical record is less complete.

The earthquakes are separated into general categories of strike-slip, normal, and reverse faulting. Figure 2 shows the exceedance rates of considered earthquakes in each of these categories as a function of magnitude, both combined and separated by focal mechanism. To estimate the rates, the number of earthquakes for each of the curves was divided by 100 years. This is obviously an approximation, but considering Figure 1, the events prior to ~1910 may roughly compensate for the missing events since 1910. For instance, Figure 2 suggests that continental events that cause surface rupture with $M_W \geq 7.0$ have occurred at a rate of about $0.5 \text{ yr}^{-1}$, or roughly once every two years. The rates of strike-slip, reverse, and normal mechanisms are about $0.4 \text{ yr}^{-1}$, $0.075 \text{ yr}^{-1}$, and $0.045 \text{ yr}^{-1}$. Rounded to the nearest 5% this implies that about 75% of those events were strike slip, about 15% had reverse mechanisms, and about 10% had normal mechanisms.
Figure 3 shows maps with locations of all events, using different symbols to distinguish among mechanisms. The insets show more details on locations of events from the western US, the eastern Mediterranean region, and Japan.

Table 1: Earthquakes from 1968-2011 used in this study.

<table>
<thead>
<tr>
<th>Event Number</th>
<th>Event Name</th>
<th>Event Date</th>
<th>Mw</th>
<th>Rupture Length, km</th>
<th>Slip Rate, mm/yr</th>
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<td>5</td>
<td>El Mayor Cucapah</td>
<td>4-Apr-2010</td>
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<td>117</td>
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</tr>
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<td>7</td>
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<td>8-Oct-2005</td>
<td>7.6</td>
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</tr>
<tr>
<td>8</td>
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Table 2: Earthquakes from 1848 - 1967 used in this study.

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<th>Event Number</th>
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<th>Event Date</th>
<th>Mw</th>
<th>Rupture Length, km</th>
<th>Slip Rate, mm/yr</th>
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<td>28</td>
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<td>Alake Lake or Dulan</td>
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<td>7</td>
<td>40</td>
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Figure 1: Cumulative number of events used in this analysis (Table 1 and 2), shown as a function of time.

Figure 2: Event rates, as a function of magnitude and event types. The rates are estimated based on the approximation that the data represent about 100 years of seismicity, as discussed in the text.
Modeling Approaches

The effect of slip rate is tested against three model shapes for the scaling relationship to confirm that it is not an artifact of a particular assumption for how magnitude depends on rupture length. The first, M1, explores a linear regression of Mw with the logs of length and slip rate:

\[ M_W = c_0 + c_1 \log L_E + c_2 \log \frac{S_F}{S_0} \]  

(1)

where \( L_E \) is the rupture length (measured along strike) of a specific earthquake, \( M_W \) is the reported moment magnitude for the respective earthquake (Kanamori, 1977), \( S_F \) is the slip rate of the fault on which the earthquake oc-
curred determined from geological observation, $S_0$ is the average of the logs of all slip rates in the data set being considered (e.g. strike-slip faults, normal faults, etc), and $c_0$, $c_1$, and $c_2$ are coefficients of regression to be determined. Mathematically, $c_0$ trades off with $-c_2 \log S_0$, which allows the parameter $S_0$ to be rounded to two significant digits. In this model, setting $S_F = S_0$ is mathematically equivalent to setting $c_2 = 0$, and thus also equivalent to the model approach used by Wells and Coppersmith (1994) and others who estimate a linear dependence of $M_W$ on $\log L_E$ without including the slip rate on the fault.

Two misfit parameters are considered. The first, $\sigma_{1L}$ is the standard deviation of the difference between observed and predicted magnitudes when $c_2 = 0$, so only $L_E$ is used to estimate $M_W$, while $\sigma_{1S}$ is the corresponding standard deviation when the slip rate term in Equation 1 is incorporated. A consequence of the assumed model M1 is that unless $c_1$ fortuitiously equals $2/3$, stress drop increases for large earthquakes as a function of rupture length $L_E$, regardless of whether slip rate is included or not (Table 6, Appendix).

The second, M2, constrains the slope to give constant stress-drop for small and large earthquakes with a slope change at the break point magnitude $M_{bp}$.

The stress drop for small and large earthquakes is allowed to differ:

\[
M_W = M_{bp} + c_{1C} \log \left( \frac{L_E}{L_{bp}} \right) + c_2 \log \left( \frac{S_F}{S_0} \right) \quad L_E < L_{bp}
\]

\[
M_W = M_{bp} + c_{1L} \log \left( \frac{L_E}{L_{bp}} \right) + c_2 \log \left( \frac{S_F}{S_0} \right) \quad L_E \geq L_{bp}
\]

where $c_{1C} = 2$ and $c_{1L} = 2/3$ for rupture lengths that are respectively less than or greater than $L_{bp}$, the rupture length where slope changes from 2 to $2/3$.

The three unknown parameters in model M2 are $L_{bp}$, the rupture length where the slope changes from 2 to $2/3$, $M_{bp}$, the magnitude at that transition, and $c_2$ which is again the sensitivity of magnitude to fault slip rate. Ruptures of length less than $L_{bp}$ are considered to be a small earthquake, and scale like a
circular rupture in Table 6 of the Appendix, implying that constant stress drop occurs when $c_{1C} = 2$. An earthquake with rupture of length greater than $L_{bp}$ is considered to be a large earthquake and corresponds to one of the models for a long fault in Table 6 of the Appendix (depending on fault mechanism), for which the value $c_{1L} = 2/3$ results in constant stress drop. However Equation 2 does not require the stress drop for the small earthquakes to be the same as the stress drop for large earthquakes. Equation 2 has the same number of unknown parameters to be determined from the data as Equation 1. The two standard deviations of the misfit for Model M2 are $\sigma_{2L}$ and $\sigma_{2S}$ which correspond directly to the parameters $\sigma_{1L}$ and $\sigma_{1S}$ of model M1.

The third model, M3, is derived from the model of Chinnery (1964) for a vertical strike-slip fault that ruptures the surface. It is assumed that stress drop for the top center of the fault in this model, $\Delta \tau_C$, is constant across all rupture lengths and magnitudes:

$$
M_W = \begin{cases} 
2 \log L_E + \frac{2}{3} \log \Delta \tau_C + \frac{2}{3} \left( \log \frac{2 \pi}{C_{LW} C(\gamma)} - 16.1 \right) + c_2 \log \left( \frac{S_F}{S_0} \right) & \frac{L_E}{C_{LW}} < W_{max} \\
\frac{2}{3} \log L_E + \frac{2}{3} \log \Delta \tau_C + \frac{2}{3} \left( \log \frac{2 \pi W_{max}^2}{C^2(\gamma)} - 16.1 \right) + c_2 \log \left( \frac{S_F}{S_0} \right) & \frac{L_E}{C_{LW}} \geq W_{max}
\end{cases}
$$

where

$$
C(\gamma) = 2 \cos \gamma + 3 \tan \gamma - \frac{\cos \gamma \sin \gamma (3 + 4 \sin \gamma)}{(1 + \sin \gamma)^2}
$$

Details on the development of the model M3 equations are provided in the Appendix. The value $\gamma$ is the angle from the top center of the fault to either of the bottom corners, i.e. $\tan \gamma = 2W_E/L_E$ where $W_E$ is the downdip width of the earthquake rupture. The model variables include four parameters. These are the aspect ratio of the fault for small ruptures, $C_{LW} = L_E/W_E$, the stress drop, $\Delta \tau_C$, the coefficient that quantifies the slip rate dependence, $c_2$, and
the maximum fault width $W_{\text{max}}$. Equation 3 assumes that the aspect ratio is constant for small earthquakes, and that when the selected aspect ratio in combination with $L_E$ implies a width greater than $W_{\text{max}}$ the width is set to $W_{\text{max}}$. For model M3, the two standard deviations of the misfit are $\sigma_{3L}$ and $\sigma_{3S}$, corresponding to the parameters $\sigma_{1L}$ and $\sigma_{1S}$ of model M1. As written, the coefficients of the term in $\log L_E$ appear to be the same as in model M2, but for the long ruptures, $\gamma$ depends on $L_E$, so the term with $C(\gamma)$ modifies the slope.

Model Equations 1, 2, and 3 require different strategies to obtain their unknown coefficients. The simplest way to find the unknown coefficients for Equation 1 is by using a linear least-squares regression, which minimizes the misfit of the prediction of $M_W$, but does not account for uncertainties in $L_E$ or $S_F$. AWS96 approached this difficulty by carrying out multiple regressions for points chosen at random within the range of allowed values of all three parameters, and then looked at the distribution of derived values of the coefficients of the regression. Alternative approaches to find the coefficients, described variously as “total least squares” or “general orthogonal regression” (e.g. Castellaro et al., 2006; Castellaro and Borman, 2007; Wikipedia article “Total Least Squares” accessed Feb. 15, 2015) were also considered for this analysis. The approach by AWS96 turned out to give the least biased results for a set of synthetic data with an uncertainty model that we considered to be realistic and consistent with the actual data, so their approach is also used in this study. The parameters for Equation 1 were determined from 10,000 realizations of the randomized earthquake parameters to find the distributions of coefficients.

In implementing the AWS96 approach, $M_W$, $L_E$, and $S_F$ are chosen at random from the range of uncertainties given in the Online Supplement. The probability distributions for the randomized parameters reflect that uncertainty
ranges are not symmetrical around the preferred value. The preferred value is set to be the median. As an example, the probability distribution for the \( i \)\(^{th} \) randomized value of \( L_E \) is:

\[
p(L_{Ei}) = \begin{cases} 
\frac{1}{(L_{E}^{\text{pref}} - L_{E}^{\text{min}})} & (A) \\
\frac{1}{(L_{E}^{\text{max}} - L_{E}^{\text{pref}})} & (B)
\end{cases}
\]

In Equation 5, case A has probability of 0.5, case B has probability of 0.5, \( L_{E}^{\text{min}} \) and \( L_{E}^{\max} \) are the minimum and maximum of the range on the rupture length, and \( L_{E}^{\text{pref}} \) is the preferred value. The seismic moment and slip rate are randomized using the same algorithm, and \( M_W \) is found from the randomized moment. The standard deviations of the misfit, \( \sigma_L \) and \( \sigma_S \), are the average values from the multiple realizations.

Equation 2 has the additional complication of being non-linear in \( L_{bp} \). We approach the solution by reorganizing Equation 2 as

\[
M_{bp} + c_2 \log \left( \frac{S_F}{S_0} \right) = M_W - c_{1x} \log \left( \frac{L_E}{L_{bp}} \right)
\]

where \( c_{1x} \) is either \( c_{1C} \) or \( c_{1L} \) depending of \( L_E \). Assuming a value for \( L_{bp} \), it is straightforward to find the unknown coefficients \( M_{bp} \) and \( c_2 \). We considered a set of closely-spaced values of \( L_{bp} \) from the smallest to the longest rupture length in the data, and choose the value with the smallest total misfit. For each trial value of \( L_{bp} \), we solved for the unknown coefficients 10,000 times with values of \( M_W, L_E, \) and \( S_F \) randomized as in Equation 5, and our preferred model is the mean of the coefficients from the multiple realizations.

Model M3 (Equation 3) has four unknown parameters, where the effects of \( C_{LW} \) and \( W_{max} \) are nonlinear (Appendix, Figure 9). For this reason, a grid of values of \( C_{LW} \) and \( W_{max} \) was searched; there were 506 points on this grid.
For each grid point, $\Delta \tau_C$ and $c_2$ were determined by linear least squares for 10,000 randomly chosen realizations of $M_W$, $L_E$, and $S_F$. The average value of $\Delta \tau_C$ and $c_2$ was found from the distributions of these realizations, together with average values of $\sigma_{3L}$ and $\sigma_{3S}$. This permitted creating a contour plots of $\sigma_{3L}$ and $\sigma_{3S}$ as a function of the trial values of $C_{LW}$ and $W_{\text{max}}$. The minima in $\sigma_{3L}$ and $\sigma_{3S}$ did not generally occur for the same combinations of $C_{LW}$ and $W_{\text{max}}$. Because the results of model M3 might potentially be used for faults where slip rate is unknown, we minimized $\sigma_{3L}$. The minimum in $\sigma_{3L}$ is broad compared to the grid spacing of $C_{LW}$ and $W_{\text{max}}$, so the values that are used come as near as possible, within the minimum of $\sigma_{3L}$, to minimize $\sigma_{3S}$ as well. The grid limits were chosen to only include maximum fault widths that were considered reasonable from a physical standpoint, while also noting the suggestions of King and Wesnousky (2007), Hillers and Wesnousky (2008) and Jiang and Lapusta (2016) that a dynamic rupture in a large earthquake might reasonably extend deeper than the brittle crustal depths associated with microearthquakes.

**Analysis Results**

Figures 4, 5, and 6 show results for models M1, M2, and M3 respectively, for strike-slip, reverse, and normal faulting earthquakes. For each mechanism, the curve in the upper frame shows predicted values of magnitude, $\hat{M}_W$, for $S_F = S_0$. The lower frame shows the residuals from this prediction, defined as

$$\Delta M_{W_i} = M_{W_i} - \hat{M}_{W_i}$$

for each considered earthquake, and the solid line is given by $\Delta M_W = c_2 \log (S_F/S_0)$. Model coefficients and uncertainties in estimates of $M_W$ for models M1, M2, and M3 are given in Tables 3, 4, and 5, respectively.
Figure 4: Models M1 for (A) strike-slip, (B) reverse, and (C) normal faults. For each mechanism, the upper frame shows $M_W$ plotted as a function of $L_E$. Points are all the preferred values, as given in Tables 1 and 2. Solid points represent low slip-rate faults. The solid line uses coefficients given in Table 3 for $S_F = S_0$. The lower frame shows the residuals, $\Delta M_W$, of the points in the upper frame from the solid line. The line in the lower frame shows the predicted effect of $S_F$ based on the coefficients in Table 3, i.e. $\Delta M_W = c_2 \log (S_F / S_0)$. For strike-slip faults, the significant effect of fault slip rate is seen in the clear separation of low and high slip-rate faults in the upper panel, and the negative slope of the fit to the residuals in the lower panel. For reverse and normal faults, the sparse data suggest a different trend in the residuals, indicating that mixing the three mechanism types is not appropriate.
Figure 5: Models M2 for (A) strike-slip, (B) reverse, and (C) normal faults. Other figure details are as in Figure 4.
Figure 6: Models M3 for (A) strike-slip, (B) reverse, and (C) normal faults. Other figure details are as in Figure 4.
Model M1: The linear model

The parameters for the linear models are given in Table 3. Figure 7 shows the distribution of coefficients found for 10,000 trials for strike-slip faults. The widths of these distributions are used to estimate the uncertainty in each coefficient. The coefficients $c_0$, $c_1$, and $c_2$ are found simultaneously, as opposed to a possible alternative approach in which $c_0$ and $c_1$ could be found first, and then $c_2$ determined by a second independent linear fit to the residuals.

For strike-slip events, which dominate the data, $c_2 = -0.198 \pm 0.023$ (Figure 4A) so $\Delta M_W$ is observed to be a decreasing function of slip rate, similar to AWS96. The data with a reverse mechanism support $\Delta M_W$ increasing, rather than decreasing, with increased slip rate (Figure 4B), while for the events with a normal mechanism the slip-rate dependence of $\Delta M_W$ is not distinguishable from zero (Figure 4C). Considering the distribution of slip rate data for reverse faults in Figure 4B, it may be observed that the finding of slip rate dependence is the result of mainly a single outlier, the Marryat earthquake (M5.8, #28 in Table 1) which is reported to have a slip rate of 0.005 mm/yr. Intracontinental events are included considering, based on Byerlys Law, that the physics of rupture of crystalline rocks within the range of typical crustal compositions is not, a-priori, different merely because the fault is located far from a plate boundary or that rock type might be different (e.g. Byerly, 1978; Scholz, 2002). Also, the Marryat Creek event tends to decrease the slip rate dependence of $\Delta M_W$, as a consideration of the remaining points would reveal. Nonetheless, the positive slope of $\Delta M_W$ in Figure 4B for reverse faulting is not a robust result.

Considering Figure 4, the results for the linear model provide an indication that it is not appropriate to combine different fault types in this type of regressions. The AWS96 model from all rupture types was $M_W = 5.12 + 1.16 \log L_E - 0.20 \log S_F$, which is only slightly different from the strike-slip case in Figure
4A. That result is consistent with the AWS96 model being dominated by strike-slip earthquakes, and thus demonstrates continuity with the previous study. However, results here separated by mechanism indicate that the slip-rate dependence in AWS96 is controlled by the behavior of strike slip earthquakes, and not much affected by the normal mechanisms that show little or no slip-rate dependence, and the reverse mechanisms that potentially show a different dependence. Suppose as a thought experiment that the dip-slip cases have no slip rate dependence, or in other words that the variability with slip rate is pure noise. A strong strike-slip case plus some noise will still resolve to a decently significant trend even though we added only noise. In applying the combined regression to dip-slip faults, we may be projecting back from the strong case into the noise, and saying things about future dip-slip earthquake expectations that aren’t likely based on the available data.

Table 3: Coefficients for Model, M1, for use in Equation 1, for the different fault types considered separately, for earthquakes listed in Table 1 and 2.

<table>
<thead>
<tr>
<th></th>
<th>Strike-Slip</th>
<th>Reverse</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>4.73 ± 0.062</td>
<td>5.11 ± 0.11</td>
<td>5.25 ± 0.18</td>
</tr>
<tr>
<td>$c_2$</td>
<td>1.30 ± 0.031</td>
<td>1.15 ± 0.065</td>
<td>1.02 ± 0.12</td>
</tr>
<tr>
<td>$c_3$</td>
<td>-0.198 ± 0.023</td>
<td>0.264 ± 0.036</td>
<td>-0.116 ± 0.109</td>
</tr>
<tr>
<td>$S_0$</td>
<td>4.8 mm/yr</td>
<td>1.1 mm/yr</td>
<td>0.25 mm/yr</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>0.242</td>
<td>0.322</td>
<td>0.318</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>0.211</td>
<td>0.238</td>
<td>0.303</td>
</tr>
</tbody>
</table>
Figure 7: Coefficient distributions for the linear, strike-slip case. The bar chart shows number of occurrences of parameter values among 10,000 realizations for randomly selected values of Mw, LE, and SF within the uncertainty range of each. Solid gray line shows the mean value of each parameter. The dashed grey line shows the value found for the preferred value of Mw, LE, and SF for each earthquake. The clear negative value of c2 corresponds to decreasing relative magnitude predictions with increasing slip rate.

Model M2: The bilinear model

Model M2 is calculated following the approach described in Equation 6. Table 4 gives estimated coefficients. Without the slip-rate adjustment, the bilinear model fits the observed magnitudes as well or better than the linear model M1, as shown by similar or smaller values of $\sigma_{2L}$ than the corresponding values of $\sigma_{1L}$. The results again show a dependence of magnitude on slip rate for strike-slip faults but not dip-slip faults (Figure 5). The value of $\sigma_{2S}$ is comparable to $\sigma_{1S}$ for the strike-slip case, but larger for the dip-slip faults. For the strike-slip case,
the fit to the data in Figure 5A is better at large rupture lengths than in Figure 4A.

Table 4: Coefficients for Model M2, for use in Equation 2, for the different fault types considered separately, for earthquakes listed in Table 1 and 2.

<table>
<thead>
<tr>
<th></th>
<th>Strike-Slip</th>
<th>Reverse</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{bp}$</td>
<td>73.9± 9.4</td>
<td>46.4± 6.4</td>
<td>24.3 ± 1.10</td>
</tr>
<tr>
<td>$M_{bp}$</td>
<td>7.38± 0.070</td>
<td>7.23± 0.091</td>
<td>6.80± 0.031</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-0.176± 0.031</td>
<td>0.170± 0.042</td>
<td>-0.106± 0.091</td>
</tr>
<tr>
<td>$S_0$</td>
<td>4.80</td>
<td>1.1</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_{2L}$</td>
<td>0.238</td>
<td>0.281</td>
<td>0.289</td>
</tr>
<tr>
<td>$\sigma_{2S}$</td>
<td>0.212</td>
<td>0.253</td>
<td>0.277</td>
</tr>
</tbody>
</table>

Model M3: The constant stress drop model

Parameters for model M3 are given in Table 5, and the fit to the data is illustrated in Figure 6. Some features of Figure 6 are noteworthy. For the strike-slip case, the points for faults with low slip rates (solid points) are mostly above the average model, while points with high slip rates (open circles) are mostly below the average. This slip-rate dependence is reinforced in the lower frame of Figure 6A, where the slope of the linear fit to the residuals is more than five standard deviations of the slope different from zero. The variance reduction by the addition of the slip rate term is statistically significant with 80% confidence, based on the F-test. The same remarks apply for models M1 (Figure 4) and M2 (Figure 5).

For strike-slip cases, both $\sigma_{3L}$ and $\sigma_{3S}$ are smaller than the equivalent uncertainties in models M1 or M2. While this improvement is small, it is encouraging that a model with constant stress drop achieves this result. $W_{max}$ for the strike-slip case is 20 km, which is in the range of up to 25 km considered by King and Wesnousky (2007). The optimal aspect ratio found here is 2.9.

For the reverse faulting data we considered values of $W_{max}$ up to 30 km since reverse faults can have low dips, and that upper limit is the preferred value. For
normal faulting, we only considered values of $W_{max} > 18$ km, as constrained observations of normal faults imply that the fault width can be that wide (e.g. Richins et al., 1987). For M3 to fit the sparse normal-faulting data as well as M2, we would need to use a much smaller value of $W_{max}$.

Table 5: Coefficients for Model M3, for use in Equation 3, for the different fault types considered separately, for earthquakes listed in Table 1 and 2.

<table>
<thead>
<tr>
<th></th>
<th>Strike-Slip</th>
<th>Reverse</th>
<th>Normal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \tau_C$, bars</td>
<td>30.6±1.3</td>
<td>21.3±1.4</td>
<td>28.3±3.0</td>
</tr>
<tr>
<td>$C_{LW}$</td>
<td>2.9</td>
<td>1.4</td>
<td>1.2</td>
</tr>
<tr>
<td>$W_{max}$, km</td>
<td>20</td>
<td>30</td>
<td>18</td>
</tr>
<tr>
<td>$c_2$</td>
<td>-0.175±0.029</td>
<td>0.144±0.027</td>
<td>0.055±0.095</td>
</tr>
<tr>
<td>$S_0$, mm/yr</td>
<td>4.8</td>
<td>1.1</td>
<td>0.25</td>
</tr>
<tr>
<td>$\sigma_{3L}$</td>
<td>0.235</td>
<td>0.281</td>
<td>0.312</td>
</tr>
<tr>
<td>$\sigma_{3S}$</td>
<td>0.210</td>
<td>0.255</td>
<td>0.305</td>
</tr>
</tbody>
</table>

Comparisons

Finally, Figure 8 compares the models for the three different types of mechanism. Models M2 and M3 tend to resemble each other most closely, while model M1, being linear, tends to give larger magnitudes for long and short faults, but smaller magnitudes in the center of the length range.
Figure 8: Comparisons of models M1, M2, and M3. The plots compare M1, M2, and M3 for strike-slip, reverse, and normal faults, using $S_F = S_0$ for each mechanism.

**Discussion**

The larger data set modeled here compared to AWS96 expands our understanding of slip rate dependence for the scaling of magnitude and rupture length. Improvements in estimates of the magnitudes of earthquakes are realized with slip rate dependence for strike-slip faults for all three models considered here. Thus the slip-rate dependence in this case is not an artifact of the underlying scaling model. On the other hand, individual models for reverse and normal faulting have, at best, an equivocal place for slip rate dependence. For linear and bilinear models of the normal faulting events, but not the constant stress drop model, at least the sign of slip rate dependence agrees with strike-slip.
Thus normal faulting could have slip rate dependence nudging estimates toward smaller magnitudes for higher slip rate faults, but lack sufficient data to prove it. The reverse faulting events disagree even at the sign of the effect. The disagreement is present whether we retain either or both of the apparent outliers in Figures 4-6. Thus based on the current data, we do not find support for the general reduction of magnitude with slip rate implied by the combined set regression. It appears that the strength of the slip rate effect among strike slip events and their shear numbers relative to dip slip events overwhelm the ambiguous (normal) and contrary (reverse) data, leading to an apparently general slip-rate relationship among all data. Thus our new data set contributes the understanding that slip rate dependence is dominantly a strike-slip fault effect that is not inconsistent with normal faulting, and not apparently consistent with reverse mechanism fault rupture. The data available to AWS96 did not permit this distinction.

If we are guided by studies of earthquake source physics, model M3 may be preferable to M1 or M2. Specifically, the advantage would be the constant stress drop of earthquakes over the full range of magnitudes, consistent with e.g. Allmann and Shearer (2009). The slope of the linear model, M1, with rupture length implies that stress drop increases significantly with rupture length for large earthquakes. The slopes of the bilinear model, M2, are consistent with simple models for scaling with constant stress drop in the small and large earthquake domains, but the stress drops in the two domains are different. In addition, since the buried circular rupture model by construction does not reach the surface, its applicability to the short ruptures of M2 is not obvious.

Constant stress drop model M3 has the important advantage compared to dislocation models in an unbounded space, in that it is explicitly designed for surface-rupturing earthquakes. Stress differences due to the free surface effect
of rupture are recognized as having significant effects on ground motion. For this reason we can expect that it will perform well where magnitude scaling is required for application to strong ground motions. The Chinnery (1964) model has a singularity of stress drop near its edges, which enables a closed form solution. The approximation of uniform slip with a singularity at the edges is probably no more significant than those already made to summarize spatial variability across the fault in stress drop of actual individual earthquakes with a single average value. The application of the same functional form for dip-slip earthquakes is entirely ad-hoc, of course. Although it is more complicated, its consistency with a physical model with a constant stress drop commends it as a preferred regression. Compared with the better-known equations summarized by Kanamori and Anderson (1975), the stress drop parameter in this model is smaller, emphasizing that all average stress drop estimates are model dependent.

The adjustment that decreases magnitude for high-slip rate strike-slip faults implies that the stress drop on those faults is lower than faults of the same length with lower slip rate. The finding is consistent with the observations of Kanamori and Allen (1986) and Scholz et al. (1986) that a longer healing time results in a larger stress required to initiate rupture, and thus a higher stress drop. For normal or reverse faulting, the slip-rate dependence is low, and the slip-rate coefficient \( c_2 \) is indistinguishable from zero. The findings suggest that, if \( c_2 \) is not zero for these cases, then \( c_2 \) is positive for reverse faulting earthquakes. This is contrary to the hypothesis of Kanamori and Allen. We suggest that if this positive slope is confirmed with added data, the physical mechanism may be related to the dynamics of rupture. For a reverse fault, the dynamic stresses on a rupture propagating updip are tensile as rupture approaches the surface, so the coefficient of friction or cohesion on the fault is less relevant.

There are a number of future studies that should be performed to improve
upon the results presented here. The first is to examine the consistency of
the models, and especially M3, with observed fault displacement. If the re-
results, based on the definition of seismic moment (Equation 9) agree with seismic
data, the scaling relationship presented here would be an alternative to the self-
consistent scaling model of Leonard (2010, 2014) for earthquakes in continental
crust.

A second issue that deserves attention is handling multi-segment faults. We
consider, for instance, the 1905 Bulnay, Mongolia, earthquake, which is the
strike-slip point in Figures 4, 5, and 6 at 375 km and M8.5. The 375 km
length is the distance from one end of the rupture to the other, and does not
include a spur fault in between that is 100 km long. This event points out that
the standard deviations $\sigma_{xL}$ and $\sigma_{xS}$ for all three models include the potential
presence of spur or subparallel faults that do not increase the total end-to-end
length of the rupture. Several other faults in Table 1 and 2 have similar issues. A
better understanding of how seismic moment is distributed on multiple segments
and fault splays, as well as how best to measure the lengths of multiple segment
ruptures and how to recognize these features ahead of the earthquake would
help to reduce uncertainties in future studies of scaling relations. If the result
is different from the approach used by UCERF3, it could have a direct impact
on future seismic hazard analyses.

**Conclusions**

The primary question asked by this research is if the introduction of slip rate
on a fault helps to reduce the uncertainties in estimates of magnitude from
observations of rupture length. We find that such a slip rate dependence is
reasonably well established for strike-slip cases: as the slip rate increases for
any given fault length, the predicted magnitude tends to decrease. This result
is robust in the sense that the slope of the residuals with slip rate is significantly different from zero and the variance reduction is modestly significant for all three of the considered models relating rupture length and magnitude. For reverse and normal faulting mechanisms, on the other hand, our data do not demonstrate the presence of a significant slip rate effect in the relationship between rupture length and magnitude. Compared to original results in AWS96, we now suggest slip rate be included only for strike-slip faults.

The constant stress drop model presented here has potential for progress on a standing difficulty in ground motion modeling of an internally consistent scaling of magnitude, length, down-dip width, and fault displacement. Current relations in which magnitude scales with length or area lead to unphysical stress drops or unobserved down-dip widths, respectively. By working from the model of Chinnery (1964) our constant stress drop model has the advantage of starting with realistic physics including the stress effects of surface rupture. Work remains in validating displacements from our model, but the fact that it fits the current magnitude-length-slip rate data as well or a bit better than the linear and bilinear models suggests that the constant stress drop is preferable to models that do not have this feature.

Resources

The Wikipedia article “Total Least Squares” was accessed Feb. 15, 2015.

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terpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Government.

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Models M3 for (A) strike-slip, (B) reverse, and (C) normal faults. Other figure details are as in Figure 4.
Coefficient distributions for the linear, strike-slip case. The bar chart shows number of occurrences of parameter values among 10,000 realizations for randomly selected values of Mw, LE, and SF within the uncertainty range of each. Solid gray line shows the mean value of each parameter. The dashed grey line shows the value found for the preferred value of Mw, LE, and SF for each earthquake. The clear negative value of c2 corresponds to decreasing relative magnitude predictions with increasing slip rate.

Comparisons of models M1, M2, and M3. The plots compare M1, M2, and M3 for strike-slip, reverse, and normal faults, using $S_F = S_0$ for each mechanism.

Model for $M_W$ based on Chinnery (1964) scaling as given in Equation 25. Top: effect of changing the stress drop $\Delta \tau_C$. Center: effect of changing the aspect ratio of the fault. Bottom: effect of changing the limiting rupture width $W_{max}$.

A Appendix: Fault scaling relations

Basics

This paper proposes models to estimate the moment magnitude of earthquakes based on observed surface rupture lengths and slip rates. The moment magnitude definition that we use is, implicit in Kanamori (1977):

$$M_W = \frac{2}{3} (\log M_0 - 16.1)$$

The units of seismic moment, $M_0$, are dyne-cm in Equation 8. This definition differs slightly from the equation used by Hanks and Kanamori (1979), but is the equation recommended for seismic network operations by the International
The seismic moment is defined as:

$$M_0 = \mu A_E \bar{D}_E = \mu L_E W_E \bar{D}_E$$

(9)

where $\mu$ is the shear modulus, $A_E$ is the fault area ruptured in the earthquake, and $\bar{D}_E$ is the average slip over that area. For a fault that is approximately rectangular, $A_E = L_E W_E$ where $L_E$ is the rupture length measured along strike, and $W_E$ is the downdip rupture width.

Substituting Equation 9 into Equation 8, one obtains (for cgs units)

$$M_W = \frac{2}{3} \log L_E + \frac{2}{3} \log W_E + \frac{2}{3} \log \bar{D}_E + \frac{2}{3} (\log \mu - 16.1)$$

(10)

This justifies models that relate magnitude to the log of fault length, width, and mean slip. Slopes different from $2/3$ result from correlations among the fault parameters $L_E$, $W_E$, and $\bar{D}_E$. Wells and Coppersmith (1996) found that the model

$$M_W = c_1 \log L_E + c_0$$

(11)

predicts magnitude from rupture length with a standard deviation of the misfit, $\sigma_1 = 0.28$.

The possible dependence of stress drop or magnitude on slip rate was recognized by Kanamori and Allen (1986) and Scholz et al. (1986). Equation 11 with the addition of the slip rate term, used by AWS96, is:

$$M_W = c_0 + c_1 \log L_E + c_2 \log S_F$$

(12)

Testing for a logarithmic dependence on the geological fault slip rate, $S_F$, can
be motivated by findings in Dieterich (1972). In this paper, we refer to Equation 12 as Model 1, or more briefly M1.

**Constant Stress Drop Scaling**

The static stress drop, $\Delta \tau_S$, is the average decrease in the shear stress acting on the fault as a result of the earthquake, and is proportional to the ratio of average slip to a fault dimension. Seismic observations have found that the average value of $\Delta \tau_S$ is approximately constant (~4 MPa, ~40 bars) over a broad range of earthquake magnitudes (e.g. Kanamori and Anderson, 1975; Allmann and Shearer, 2009), although there is considerable scatter in these data. Seismic moment, and thus $M_W$, through Equation 8, can be expressed as a function of fault dimension and stress drop, as recognized by Kanamori and Anderson (1975). Selected models are summarized in Table 6.

**Table 6: Models from Kanamori and Anderson (1975) for the relationship of fault size, stress drop, and $M_W$.**

<table>
<thead>
<tr>
<th>Case</th>
<th>$M_0$</th>
<th>Implied magnitude relations$^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Buried, circular</td>
<td>$\frac{4}{7} \Delta \tau_S R_E^3$</td>
<td>$M_W = \log A_E + \frac{2}{3} \log \Delta \tau_S + 3.0089$</td>
</tr>
<tr>
<td>2. Strike-slip, long</td>
<td>$\frac{3}{7} \Delta \tau_S W_E^2 L_E$</td>
<td>$M_W = \frac{3}{4} \log L_E + \frac{3}{4} \log W_E + \frac{2}{3} \log \Delta \tau_S + 3.1359$</td>
</tr>
<tr>
<td>3. Dip-slip, long</td>
<td>( \frac{\pi}{4(\lambda+\mu)} \Delta \tau_S W_E^2 L_E )</td>
<td>$M_W = \frac{2}{3} \log L_E + \frac{3}{4} \log W_E + \frac{2}{3} \log \Delta \tau_S + 3.3141$</td>
</tr>
</tbody>
</table>

1. Fault area, $A_E = \pi R_E^2$, in km$^2$, fault radius $R_E$, width $W_E$, and length $L_E$ in km, and stress drop $\Delta \tau_S$ in bars.

The equations in Table 6 indicate that constant stress drop implies the slope $c_1 = 2.0$ for small faults (Case 1) when $L_E$ is equated to the diameter of the circular fault and $c_1 = 2/3$ for long faults (Cases 2 and 3). These observations motivate a bilinear approach to fit the data, which is model M2 in the main text of the paper. The bilinear approach is formulated as follows:

$$M_W = M_{bp} + c_1 \log \left( \frac{L_E}{L_{kp}} \right) + c_2 \log \left( \frac{S_E}{S_0} \right) \quad L_E < L_{kp}$$
$$M_W = M_{bp} + c_1 L_E + c_2 \log \left( \frac{S_E}{S_0} \right) \quad L_E \geq L_{kp} \quad (13)$$

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In Equation 13 the length $L_{bp}$ is the length at which the length-dependence of the scaling relationship changes from the small fault model with slope $c_{1C} = 2$ to the long fault model with slope $c_{1L} = \frac{2}{3}$. The slip rate $S_0$ is a reference slip rate which can be chosen arbitrarily, but is conveniently chosen to be the log average slip rate in the data, so that setting $S_F = S_0$ gives the best fit when slip rate is unknown. The constant $M_{bp}$ is the magnitude corresponding to a fault with length $L_E = L_{bp}$ and slip rate $S_F = S_0$. Note that Equation 13 has three unknown coefficients ($M_{bp}$, $L_{bp}$, and $c_2$), which is the same number as Equation 12.

However, there are issues with the applicability of the equations 1, 2, and 3 in Table 6. The foremost, for the long faults, is the width of the seismogenic zone. Table 6 shows that $W_E$ is twice as influential as the fault length, so it needs to be considered carefully. One approach to estimate this width is to use the maximum depth of microearthquakes. By this approach, for strike-slip earthquakes the maximum depth of microearthquakes equates directly to an estimate of the fault width, while for a reverse or normal fault the dip is incorporated. The problem is that the maximum depth of seismogenic rupture in large earthquakes is difficult to observe. King and Wesnousky (2007) discuss this difficulty, and present arguments for why the downdip width might be larger in large earthquakes, at least up to some limit greater than that inferred from the depth range of small earthquakes, because rocks below the depths of microearthquakes might experience brittle failure under high strain rates. If the width increases in general for long ruptures, stress drop is no longer as high for these events because stress drop is inversely proportional to $W_E$, and furthermore the slope $c_1$ can no longer be reliably constrained by the models in Table 6. King and Wesnousky propose that this explains the proposal by Scholz (1982) that slip in large earthquakes is more nearly proportional to rupture length than
to rupture width.

Another issue is that Equation 1 of Table 6 assumes that the circular fault is confined within the Earth and thus neglects free surface effects, while by definition all of the events considered in this study rupture the surface. This motivates development of the model that is described in the next section.

Relations based on Chinnery (1964)

Chinnery (1963, 1964) calculated a stress drop for a rectangular, strike-slip fault that ruptures the surface. Unlike the circular slip model, the free surface in the Chinnery model is present for small earthquakes. His equations assume a uniform slip on the fault. Thus the stress drop is variable over the fault, and becomes singular at the edge of the fault. His equations give the stress drop on the surface at the midpoint of the rupture. Numerical solutions in Chinnery (1963) show relatively uniform stress drop over large portions of the fault. Chinnery (1963) thus suggests that the results are valid to represent the fault stress drop so long as the zone of slip fall-off is much smaller than the area of the fault. The key advantage provided by this approach is to provide a useful analytical solution.

For the rectangular fault with length $L_E$, width $W_E$, and aspect ratio $C_{LW} = L_E/W_E$, the stress drop in the Chinnery model, $\Delta \tau_C$, at the midpoint at the surface is

$$\Delta \tau_C = \frac{\mu D_E}{2\pi} C_1 (L_h, W_E)$$  \hspace{1cm} (14)$$

where

$$C_1 (L_h, W_E) = \left\{ \frac{2L_h}{aW_E} + \frac{3}{L_h} - \frac{L_h (3a + 4W_E)}{a(a + W_E)^2} \right\}$$  \hspace{1cm} (15)$$

Note that $L_h = L_E/2$, and $a = (L_h^2 + W_E^2)^{1/2}$. Observe that $C_1$ has dimensions
of $1/\text{length}$, and thus $C_1^{-1}$ is effectively the fault dimension that is used for calculating the strain. In other words, the strain change in the earthquake is $\sim \bar{D}_E C_1$. An equation for the seismic moment can be obtained by solving Equation 14 for $\bar{D}_E$ and substituting in Equation 9. The result is

$$M_0 = 2\pi \Delta \tau_C \frac{L_E W_E}{C_1 (L_h, W_E)}$$

(16)

and thus

$$M_W = \frac{2}{3} \log L_E + \frac{2}{3} \log \Delta \tau_C + \frac{2}{5} \log \frac{2\pi W_E}{C_1 (L_h, W_E)} - \frac{2}{3} \cdot 16.1$$

(17)

Additional insight into the geometrical term can be obtained by observing that $a$ is the length of the diagonal from the midpoint of the fault at the surface to either of the bottom corners. If the dip of this line is $\gamma$, then $\tan \gamma = W_E/L_h = 2/C LW$, $L_h = a \cos \gamma$, $W_E = a \sin \gamma$, and one can rewrite

$$C_1 (L_h, W_E) = \frac{1}{W_E} C (\gamma)$$

(18)

where

$$C (\gamma) = 2 \cos \gamma + 3 \tan \gamma - \frac{\cos \gamma \sin \gamma (3 + 4 \sin \gamma)}{(1 + \sin \gamma)^2}$$

(19)

Thus one can rewrite Equation 14 as

$$\Delta \tau_C = \frac{C (\gamma)}{2\pi} C \frac{\bar{D}_E}{W_E}$$

(20)

Solving Equation 20 for $\bar{D}_E$ and substituting into Equation 9 gives the moment of a vertical strike-slip fault that ruptures the surface as:

$$M_0 = \frac{2\pi}{C (\gamma)} \Delta \tau_C L_E W_E^2$$

(21)
Because $\gamma$, and thus $C(\gamma)$, depends on the fault aspect ratio, Equations 16, or 21, can be used to model a transition from small-earthquake behavior (e.g. the circular fault in Table 6) to a long-fault behavior. This paper, similar to Hanks and Bakun (2002), maintains a constant aspect ratio as the fault length increases, until that aspect ratio implies that the fault width would exceed some maximum. For longer faults, the width is set to that maximum. Before reaching that maximum, $\gamma$ and $C(\gamma)$ are constant, and

$$M_0 = \frac{2\pi}{C(\gamma)} \Delta \tau_C \frac{L_{E}}{C_{LW}} \frac{L_{E}}{C_{LW}} < W_{\text{max}}$$

(22)

For longer faults, for which the width is limited, Equation 21 becomes

$$M_0 = \frac{2\pi}{C(\gamma)} \Delta \tau_C L_{E} W_{\text{max}}^2 \frac{L_{E}}{C_{LW}} \geq W_{\text{max}}$$

(23)

In this case, as the fault length increases while width is held constant, $\gamma$ will be decreasing. For the limit of small $\gamma$ (roughly $\gamma \lesssim 25^\circ$), Equation 19 shows that $C(\gamma) \rightarrow 2$, so Equation 16 approaches

$$M_0 = \pi \Delta \tau_C L_{E} W_{\text{max}}^2$$

(24)

Equation 24 differs from Case 2 in Table 6 for the long strike-slip fault by a factor of 2 ($\Delta \tau_S = 2 \Delta \tau_C$), where the difference is due to the different boundary conditions used for the two solutions at depth.

From Equations 22 and 23, converting to magnitude, the implied scaling relationship based on the Chinnery model is

$$M_W = \begin{cases} 
2 \log L_{E} + \frac{2}{3} \log \Delta \tau_C + \frac{2}{3} \left( \log \frac{2\pi}{C_{LW} C(\gamma)} - 16.1 \right) & \frac{L_{E}}{C_{LW}} < W_{\text{max}} \\
\frac{2}{3} \log L_{E} + \frac{2}{3} \log \Delta \tau_C + \frac{2}{3} \left( \log \frac{2\pi W_{\text{max}}^2}{C(\gamma)} - 16.1 \right) & \frac{L_{E}}{C_{LW}} \geq W_{\text{max}} 
\end{cases}$$

(25)
Equation 25 will be the third model, M3, considered in this study, with the addition of a slip rate contribution, \( +c_2 \log \left( \frac{S_F}{S_0} \right) \), to the two branches of the equation. The unknown parameters in M3 are \( \Delta \tau_C \), \( C_{LW} \), \( W_{max} \), and \( c_2 \). Thus this model has four parameters to be determined, compared to three parameters in M1 and M2. Figure 9 shows the effect of the three parameters \( \Delta \tau_C \), \( C_{LW} \), \( W_{max} \) on magnitude predictions. The stress drop scales the entire curve upwards. The aspect ratio \( C_{LD} \) adjusts the level of the magnitude for short rupture lengths. The maximum width affects the curvature and how rapidly the curve approaches the asymptotic slope of \( \frac{2}{3} \log L_E \) for long rupture lengths.

Figure 9: Model for \( M_W \) based on Chinnery (1964) scaling as given in Equation 25. Top: effect of changing the stress drop \( \Delta \tau_C \). Center: effect of changing the aspect ratio of the fault. Bottom: effect of changing the limiting rupture width \( W_{max} \).
Other models and considerations

Sato (1972) overcomes the singularity introduced by Chinnery (1963, 1964) by assuming a smooth ad-hoc slip function on a finite, rectangular/elliptical-shaped fault, and for that function, calculating the average stress drop resulting from that slip function. While the results are informative for source physics studies, the major disadvantages of this approach for our application are that the fault is embedded in a whole space, and there is no analytical solution comparable to Equation 14. Rather, the geometrical factor equivalent to \( C(\gamma) \) can be computed numerically using equations in Sato (1972) or read from a figure in the paper. Considering these limitations, this model was not considered further.

Shaw and Scholz (2001) and Shaw and Wesnousky (2008) implement a numerical model for fault slip in a half space with depth-dependent friction. They examine the statistics of events that rupture the surface. These papers are interesting for the finding that large surface rupturing events also slip below the brittle crustal depths. The scaling found in the model has properties similar to the scaling in the Chinnery model. However the scaling relationship that they determine has an ad-hoc shape, and thus we preferred the analytical functional form of Equation 25, as discussed above. The physics-based solution of Chinnery was also preferred to a related constant stress drop model by Shaw (2009). This model proposes three regimes of magnitude scaling from length based on intermediate length-width-displacement scaling relations and heuristic arguments for transitions between them.

Rolandone et al (2004) found some empirical evidence that might be interpreted to support the penetration of rupture below the brittle seismogenic layer in large earthquakes. They found that the maximum depth of aftershocks of the Landers earthquake were deeper immediately after the main shock, and then
the maximum depth returned to pre-earthquake levels over the next few years. This might be explained by high strain rates in the uppermost part of the ductile crust, as high strain rates favor brittle failure. However, post-seismic strain rates in that depth range would be high even if seismic rupture of the main shock did not extend that deep, so these observations allow, but do not require, dynamic rupture below the long-term average depth of microearthquakes.

Supplemental Online Material

The supplemental material consists of a spreadsheet that includes all earthquakes considered and a list of references for all entries in that spreadsheet.

Spreadsheet: MasterEventTable_2015-10-6.xlsx
References: References_2015_10_06.docx

A conversion of these materials to pdf format is provided here.
Online Supplement for

**Fault Scaling Relationships Depend on the Average Geological Slip Rate**

by John G. Anderson, Glenn P. Biasi, Steven G. Wesnousky

This supplement includes Table A1, which is a complete summary of the data used in this study. It also includes a file that lists all references used in Table A1.
Supplemental Material (All Other Files, i.e. Movie, Zip, tar)

aMasterEventTable_2015-10-6b.xlsx