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Reply to Yan Kagan's Comment on "The Gutenberg-Richter or Characteristic Earthquake Distribution, Which Is It?"

by Steven G. Wesnousky

There are two principal aspects to Kagan's comments. He first asserts that decisions made in analysis of the data may bias the results and interpretation I put forth in Wesnousky (1994, herein referred to as W94). He then provides his own statistical model to argue that seismicity along the major strike-slip faults of southern California is adequately described by the Gutenberg-Richter distribution Log $n = a - bM$, where n = number of events of magnitude M and a and b are empirical constants. I will initially consider the specific concerns Kagan voices toward the analysis and show that none lead to a significant change in the concluding observations, interpretations, or caveats originally stated in W94. Subsequently, I show that Kagan's statistical model is flawed when examined in the context of fault mechanics. The magnitudes of earthquakes his model requires to occur on specific faults violate empirical scaling laws that relate fault rupture length, coseismic slip, and earthquake magnitude.

Criticisms of the Analysis

Seismic Zone Selection

It is generally accepted that seismicity over broad regions and, for that matter, the globe is satisfactorily described by the Gutenberg-Richter relationship (equation 1 of W94). Kagan assumes that the same distribution must describe seismicity along individual faults. However, I have previously shown that a Gutenberg-Richter distribution of earthquakes may result from a distribution of faults of different lengths and slip rates, each of which produces an earthquake of only a single size (Wesnousky *et al.,* 1983). Theoretical earthquake fault models that incorporate various distributions of frictional properties and continuum mechanics also do not require that seismicity along individual fault zones show a Gutenberg-Richter distribution of earthquake sizes (e.g., Ben-zion and Rice, 1993, 1995). Thus, the observation that regional earthquake statistics show a Gutenberg-Richter distribution is not a sufficient condition to conclude that the same distribution describes seismicity along a particular fault. Hence, when designing an experiment to understand the shape of the earthquake frequency distribution along the major fault zones of southern California, it is not appropriate to examine earthquake statistics over the entire reach of California. Rather, it is necessary to consider seismicity that is limited to the fault zone in question.

The average width of the boxes I used in W94 to define seismicity along each of the faults is about 40 km (Fig. 2 of W94). Kagan asserts that the W94 analysis is biased by the choice of box size. An examination of Figure 2 in W94 shows that the boxes encompass all seismicity that can be reasonably associated with each of the fault zones. Closer examination shows that the width of the boxes used in W94 encompass many earthquakes that do not occur directly on the fault zones. Events located off the traces of the major fault zones cannot be attributed entirely to uncertainty in event location procedures. A significant portion, if not the majority, of those events reflect internal deformation of adjacent crustal blocks and the aftershocks of major earthquakes on adjacent major faults, both possibly related in terms of the stress field, but neither of which accommodate slip on the adjacent major fault planes. For example, the box for the San Andreas fault used in W94 includes the aftershock distributions of the M 7.5 1952 Kern County, the M 6.7 1971 San Fernando, and the M 6.1 1992 Joshua Tree earthquakes. Aftershocks of the M 6.2 1987 Elmore Ranch earthquake are included within the San Jacinto box. The choice of boxes in W94 thus yields a count of earthquakes much greater than actually occurred on the major fault zones. The argument of Kagan that a larger box width should be used is unwarranted, and the use of a lesser box width would only yield changes in the data set to further support the interpretations of W94. The question of box length is incorporated into the following section.

Size and Return Time of Largest Expected Events

The estimate of return time T for the largest expected earthquakes in W94 was based on the equation

$$T = \mathbf{M}_0^e/\dot{\mathbf{M}}_0^g, \tag{1}$$

where $\mathbf{M}_0^e$ is the expected seismic moment for an earthquake on the fault and a function of fault length (Fig. 4 of W94) and $\dot{\mathbf{M}}_0^g$ is the long-term geologically assessed seismic moment rate of the fault. The seismic moment $\mathbf{M}_0$ is equal to μLWU, where μ is the shear modulus and taken here to equal 3×10^{11} dyne/cm^2, L is fault length, W is fault width (taken as 15 km), and U is the average coseismic slip occurring across the fault (Aki and Richards, 1980). Substituting $\dot{U}^g$, the geologically determined fault slip rate, for U in the same expression allows us to describe the long-term seismic moment rate $\dot{\mathbf{M}}_0^g$ for a fault. Kagan argues that the estimates of T reported in W94 may be largely underestimated and bias the interpretations because the equation does not allow for the contribution of slip by earthquakes of either lesser or greater size than $\mathbf{M}_0^e$. The consequence of a marked decrease in the return time of the maximum expected events in magnitude-frequency distributions constructed in Figures 6 to 10 of W94 would be to change the shape of the curves to a form more consistent with the Gutenberg-Richter distribution. This does not occur and is demonstrated as follows.

To address the concern that earthquakes larger than that I allowed in W94 might rupture the major strike-slip faults of southern California, I remove for the sake of discussion the assumptions of fault segmentation used in W94 and allow that each of the faults may rupture in their entirety. In addition, I modify W94 to consider (1) the possibility that the entire 1000-km length of the San Andreas fault zone may rupture in a single earthquake and (2) the Newport-Inglewood and Rose Canyon fault zones comprise a single fault zone 200 km in length, which is also capable of rupturing in a single earthquake. The total lengths of the San Jacinto, Elsinore, and Garlock faults are 230, 240, and 240 km, respectively. In this manner, the limits of the San Andreas fault are marked by a spreading center to the south, the Brawley seismic zone, and the Mendocino fracture zone to the north, which strikes at near right angles to the San Andreas. The western end of the Garlock is truncated at high angle by the San Andreas, and the eastern limit of the fault simply ends with no continuation. At their northern ends, both Newport-Inglewood-Rose and Elsinore faults intersect at high angles to the reverse faults of the Transverse Range. To the south, the Newport-Rose Canyon fault is cut at right angles by the Agua Tibia and Garcia faults and separated by more than 10 km from the Vallecitos fault zone in Baja California to the south. The Elsinore is separated from the Laguna Salada fault to the south by a 10-km step, a reversal in the fault dip, and a change from pure strike-slip along the Elsinore to oblique dip-slip along the Laguna-Salada. Because the endpoints of these fault zones are not only marked by major geometrical discontinuities but also by major tectonic boundaries that mark fundamental changes in the style of deformation, there is little to no support to suggest that these faults may be associated with single-event ruptures extending beyond the lengths being considered here. Moreover, if one were to argue longer fault lengths than those suggested here, changes in estimated magnitudes would be relatively small (Fig. 4 of W94). For example, an assumption that the Elsinore is capable of rupturing an additional 100 km through the Cerro Prieto fault and into the Gulf of Mexico spreading center would only yield an increase in estimated maximum magnitude of 0.2 magnitude units to less than M 7.9. If Kagan's argument rests upon yet longer and, hence, larger ruptures occurring along these fault zones, it seems incumbent that he describe specifically the dimensions and locations of the type of ruptures he is forecasting along each of the fault zones, something he does not do in his comment. Otherwise, he provides no basis to examine his assertion that choice of box size may have biased the results of W94.

Toward explicitly incorporating the contribution of earthquakes smaller than the maximum expected size, I re-express equation (1) to estimate the recurrence interval T of maximum expected events along a fault zone by dividing the cumulative seismic moment release $\Sigma\mathbf{M}_0$ of all earthquakes expected during the recurrence interval T by a geologically determined average seismic moment rate $\dot{\mathbf{M}}_0^g$ for the fault

$$T = \sum\mathbf{M}_0/\dot{\mathbf{M}}_0^g = (\mathbf{M}_0^e + \sum\mathbf{M}_0^{sm})/\dot{\mathbf{M}}_0^g, \qquad (2)$$

where $\mathbf{M}_0^e$ is defined previously and $\Sigma\mathbf{M}_0^{sm}$ is the sum of seismic moment release of events with $\mathbf{M}_0 < \mathbf{M}_0^e$, which will contribute to fault slip during the recurrence interval T. Equation (3) can be rewritten as follows:

$$T = (\mathbf{M}_0^e/\dot{\mathbf{M}}_0^g)/[1 - (\dot{\mathbf{M}}_0^{sm}/\dot{\mathbf{M}}_0^g)], \qquad (3)$$

whereby $\dot{\mathbf{M}}_0^{sm}$ is approximated by the empirically determined instrumental moment release rate $\dot{\mathbf{M}}_0$ (instr) along the respective fault zones.

Using equation (3), I have recalculated the magnitude frequency distributions for each of the faults shown in Figures 6 through 10 of W94. The results are shown in Figure 1. The results are virtually identical to those shown in W94. For each fault, the preferred estimate of return time of the largest expected event falls well above the extrapolation of the maximum-likelihood fit to the instrumental record of seismicity. It is only along the San Jacinto where the estimates of return time for the maximum-sized events approach the extrapolation of the b-value curves. Therefore, when relaxing the constraint of fault segmentation and empirically incorporating the seismic moment rate of events smaller than the maximum-expected event, I arrive at the same result as stated in W94: the distributions along the Newport-Inglewood-Rose, Whittier-Elsinore, Garlock, and San Andreas are not consistent with the Gutenberg-Richter distribution.

The same result is reached if it is assumed that seismicity along the faults satisfy the Gutenberg-Richter relationship up to the limiting maximum-expected magnitude earthquake. The assumption that seismicity along the faults satisfies the Gutenberg-Richter relationship is analogous to assuming multiple rupture scenarios for the largest-expected earthquakes along each of the faults. Using the same limits on maximum magnitude and slip rate as above, and the value of $b = 1$ that Kagan considers most relevant, I calculate the expected number of magnitude 4 events for each fault and compare it in Figure 2 with the observed number. The calculations are made with a truncated magnitude-frequency distribution (equation II.5 of Anderson and Luco, 1983). The ratios of predicted-to-observed values should equal 1 if seismicity along the individual faults satisfies the Gutenberg-Richter distribution. They do not. Rather, the number of earthquakes of magnitude 4 predicted by the assumption of a Gutenberg-Richter distribution is systematically greater than observed and, hence, consistent with the idea that the

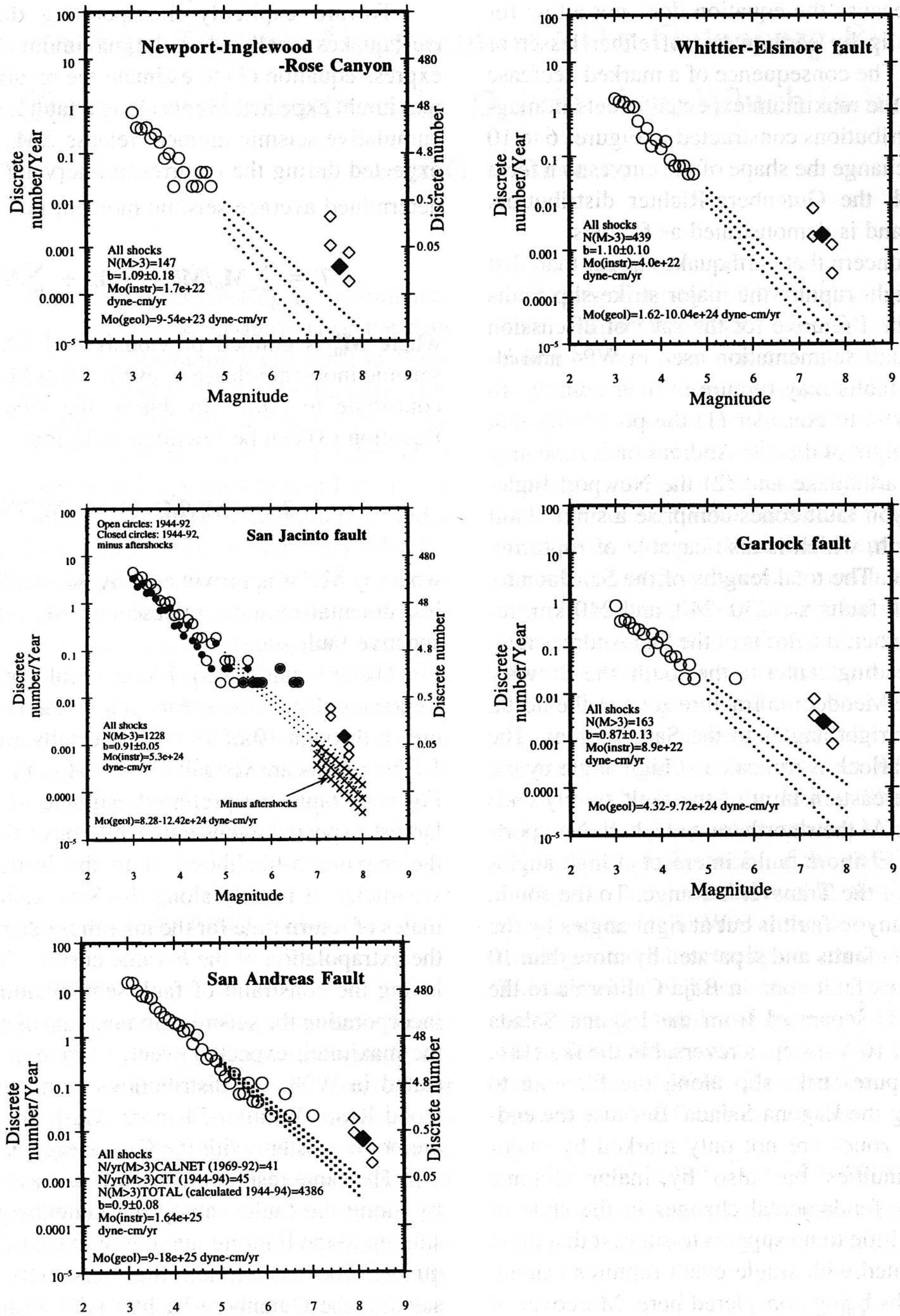

Figure 1. Discrete number of events per year versus magnitude for the Newport-Inglewood-Rose Canyon, Whittier-Elsinore, San Jacinto, Garlock, and San Andreas faults. Open circles represent the instrumental record of seismicity for the period 1944 to 1992. The dotted lines represent the 95% confidence limits on fit of the data for the period 1944 to 1992 to the equation $\log n = a - bM$, where n equals the number of events per year greater than or equal to magnitude M. The value b is determined via the maximum-likelihood method and the productivity a is defined by the total number of events recorded during the period. The total number of events (N), the b value and 95% confidence limits, and the seismic moment rate for the period 1944 to 1992 are listed in the lower left of each plot. Also listed in the lower left of each plot are the bounds on seismic moment rate determined from the fault slip-rate data ($\dot{M}_0$ (geologic)). Preferred (solid diamonds) and bounding estimates (open diamonds) of magnitude and recurrence rate of largest-expected earthquakes are determined with equation (3).

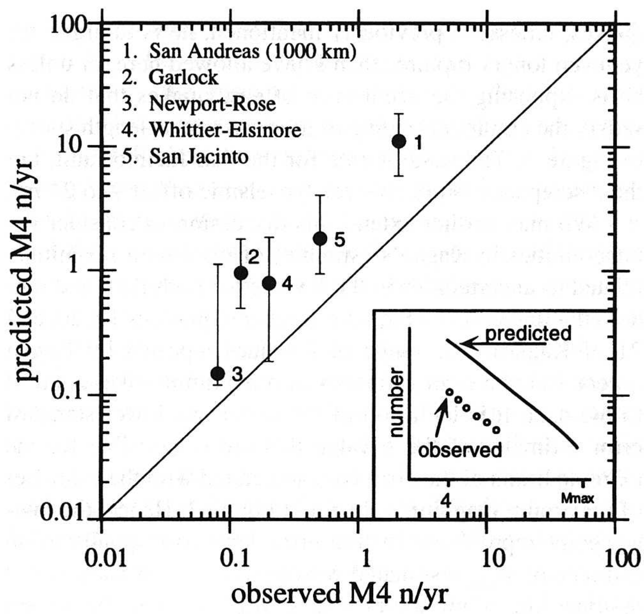

Figure 2. Discrete number of smaller events (M 4) observed on major southern California faults versus number predicted with assumption that seismicity satisfies the Gutenberg-Richter distribution up to a limiting maximum magnitude. Inset schematically illustrates the values being compared for each fault.

distribution of seismicity along the faults is more closely described by the characteristic earthquake model.

Kagan's Model

Physical Consequences

Kagan does not study fully the physical consequences attendant to his model. It is important that they be examined, for they point to a serious flaw in his approach. Anderson (1979) and Anderson and Luco (1993) showed that, for a region, given (1) an estimate of the b value, (2) the occurrence rate of earthquakes above some cutoff magnitude, and (3) knowledge of the regional rate of seismic moment release, one may estimate the maximum magnitude M_{max} earthquake needed to satisfy the regional rate of seismic moment release (e.g., see Tables 2, 3, and 4 of Anderson and Luco, 1983). Kagan cites an earlier article (Kagan, 1994) in which he followed this approach in an analysis of global seismicity to arrive at an $M_{max} = 8.0$ to 9.0. That is to say, the moment-release budget of the earth does not require earthquakes of magnitude greater than 8.0 to 9.0. He then applies the same analysis (his equation 18) to the seismicity recorded within the boxes that I defined along the major fault zones of southern California. His approach yields the estimates of M_{max} shown in Table 1 for each of the fault zones. The values reflect the maximum-likelihood estimate of b value and maximum and minimum limits on slip rates reported for the individual fault zones. Because the estimates of maximum magnitude in Table 1 generally overlap with the estimate

Table 1
Estimates of Maximum Magnitude*

Fault	Maximum Magnitude
Elsinore	8.7–10.6
Garlock	8.6–9.2
Newport-Rose	9.4–11.3
San Jacinto	7.8–8.1
San Andreas	8.7–9.2

*The values for the San Andreas and the Newport-Rose faults have been computed using Kagan's equation (18) and take into account the longer fault length assumed for this reply. Bounds on slip rate for the San Andreas and Newport-Rose faults are taken between 20 and 40 cm/yr (Petersen and Wesnousky, 1994) and 1.0 and 6.0 mm/yr (Rockwell *et al.*, 1991).

from his global study, Kagan concludes that the Gutenberg-Richter relationship cannot be rejected and, hence, is a satisfactory description of the seismicity along each of the fault zones. A requirement of his interpretation is that each fault is capable of producing the size of the earthquakes listed in Table 1.

Figure 3 is a plot of seismic moment versus fault length. The open circles are a global compilation of earthquake data published by Romanowicz (1992). The data form an empirical scaling law that shows seismic moment scales with fault rupture length in a systematic manner. The data set is limited to strike-slip earthquakes in tectonic environments similar to California, where the base of the seismogenic zone is about 15 km. The solid circles are Kagan's estimates (Table 1) of maximum magnitude that are required if seismicity along each of the fault zones is to satisfy the Gutenberg-Richter relationship. The values of moment are plotted as a function of the entire mapped length of each of the faults.

The principal observation arising from Figure 3 is that the range of maximum seismic moments determined from Kagan's analysis generally fall well above the field of historical data. Hence, the earthquakes he is predicting would be of a kind not previously seen in nature. More specifically, it indicates that the lengths of the faults are insufficient to produce the size of earthquakes he requires for his model. On that basis, his model is not viable. We can further look at the problem in terms of the amount of coseismic slip required to occur on the faults if in fact the proposed earthquakes were to occur. The average amount of coseismic slip that would be required to produce the maximum-magnitude earthquakes predicted by Kagan are listed in Table 2 for the respective faults. The median range of values for the Elsinore, Newport-Rose, Garlock, and San Andreas faults require earthquakes with coseismic offsets averaging greater than 100 m. I think this is a bit unrealistic. For comparison, the largest average coseismic offset recorded in the data set of Romanowicz (1992) is about 4 m. Thus, although Kagan can mathematically model the distribution of seismicity along these faults according to the Gutenberg-Richter distribution, his results are inconsistent with the physics of the faulting process as manifested in the global record of earth-

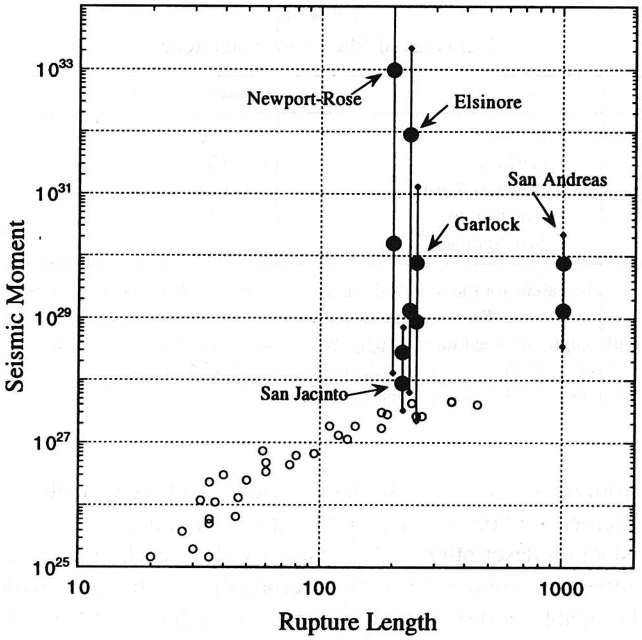

Figure 3. Seismic moment versus fault rupture length. Open circles represent the global compilation of strike-slip faults from Romanowicz (1992) and show that seismic moment scales predictably with fault length. The solid circles are the maximum magnitudes required in Kagan's model if seismicity along each of the major strike-slip faults in southern California is assumed to satisfy the Gutenberg-Richter relationship. The values are calculated using equation (18) of Kagan's comment assuming the maximum-likelihood *b* value for the individual fault zones and the minimum and maximum reported slip rates for each fault zone. Hence, there are two solid circles plotted for each the Newport-Rose, Elsinore, San Jacinto, Garlock, and San Andreas faults. The error bars further incorporate the 95% confidence limits on estimates of *b* value for each of the fault zones (equation 21 of Kagan's comment).

Table 2
Estimates of Coseismic Slip*

Fault	Length (km)	Coseismic Slip
Elsinore	240	131 m–92 km
Newport-Rose	200	1.8–1250 km
Garlock	240	93–735 m
San Jacinto	230	9–24 m
San Andreas	1000	31–176 m

*The amount of coseismic slip U is equal to $M_0/\mu LW$. For the calculations, I have converted the maximum magnitudes listed in Table 1 to seismic moment using the relationship Log M_0 = 1.5 m + 16.1 and assumed the rupture length listed, a fault width to equal 15 km, and the crustal rigidity to equal 3×10^{11} dyne/cm².

quakes, unless, as previously mentioned, he is to argue for yet even longer ruptures than I have allowed here, or unless he is supposing the occurrence of earthquakes that do not satisfy the empirical scaling of moment versus length shown in Figure 3. The same is true for the San Jacinto fault, but the discrepancy is not so large (coseismic offset 9 to 24 m).

We may further extend this discussion to consider the uncertainties in Kagan's estimates of maximum magnitude related to uncertainties in the *b* value for each fault and random fluctuation in earthquake number (equations 19, 20, and 21 of Kagan). The range of *b* values reported by Kagan reflect 1 sigma error estimates of a maximum-likelihood fit to the data. It is inclusion of the upper and lower standard error estimates of the *b* value that are responsible for the extreme limits of the error bars associated with the estimates of maximum magnitude shown in Figure 3. Hence, the lowest point represented by the error bars corresponds to an estimate of M_{max} (estimated with equation 18 of Kagan) that assumes the minimum-bounding slip rate and the lowest limit on the *b* value and conversely for the highest points of the error bars. The uncertainty bars resulting from use of equation (18) may be viewed in either of two ways. First, they are so broad as to suggest that the statistics used by Kagan are essentially uninformative. Secondly, it may be observed that it is only the extreme lower limits of the uncertainty bars that overlap with the historical data. If it is Kagan's desire to argue that this overlap is significant, his argument is based on the very fortuitous coincidence that the "correct" slip rate and *b* value are represented by the minimum-bounding slip rate reported by geologists and the minimum limit on *b* value for each of the faults considered (excluding maybe the San Jacinto). Basing support for a model on such a coincidence is, at best, not convincing. Furthermore, if the coincidence was to be accepted, we are left with an internal inconsistency in Kagan's model. His argument that the magnitude-frequency distribution along faults satisfies the Gutenberg-Richter distribution is based on the premise that his estimates of M_{max} for each of the faults (Table 1) overlap with the worldwide estimate of M_{max} = 8.0 to 9.0. However, the lower limits of the uncertainty bars for several of the events represent earthquakes smaller than the *M* 8.0 and, hence, are in conflict with the very foundation of his own arguments.

Discussion and Conclusion

In short, Kagan's model appears flawed when viewed in the context of fault mechanics. In addition, I have shown that there is no basis to argue that the analysis of W94 was biased by data selection techniques when viewed within the context of the caveats and conclusions that I put forth in W94; specifically, when combining geological data bearing on fault slip rate with a fault model based on the concept of elastic rebound, the characteristic earthquake model appears to generally be a better descriptor of seismicity along the major strike-slip faults of California. But the result should not be over interpreted. The term characteristic earthquake model has often been used to suggest that faults are characterized by the repeated occurrence of identical earthquakes. That extreme interpretation is not being argued here or in W94. Rather, the term characteristic earthquake model is only being used to encompass fault behavior whereby extrapolation of statistics historical earthquakes of small to moderate size will underestimate the occurrence of the largest-expected earthquakes along a particular fault zone. The largest-expected earthquakes do not need to be identical in size nor repeat in the exact same locations through time.

It also is not my contention that all faults necessarily behave according to the characteristic earthquake distribution. To the contrary, it seems naive to expect that all faults behave exactly according to either the characteristic earthquake or the Gutenberg-Richter model. The data for the San Jacinto fault are a possible case in point. More likely there is a continuum of behaviors, a continuum that may reflect the structural complexity of faults (e.g., Wesnousky, 1988, 1990). A study similar to W94, but much more extensive in scope, is consistent with the idea that faults characterized by complex traces tend to exhibit magnitude-frequency distributions more consistent with the Gutenberg-Richter distribution, whereas faults characterized by relatively smooth fault traces exhibit distributions more consistent with the characteristic earthquake model (Stirling *et al.,* 1995).

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Center for Neotectonic Studies
Department of Geological Sciences
University of Nevada
Reno, Nevada 89557

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